Dispersion in granular media: rheology, diffusion, segregation

Pierre Jop

Glass surface and interfaces
CNRS/ Saint-Gobain Research Paris

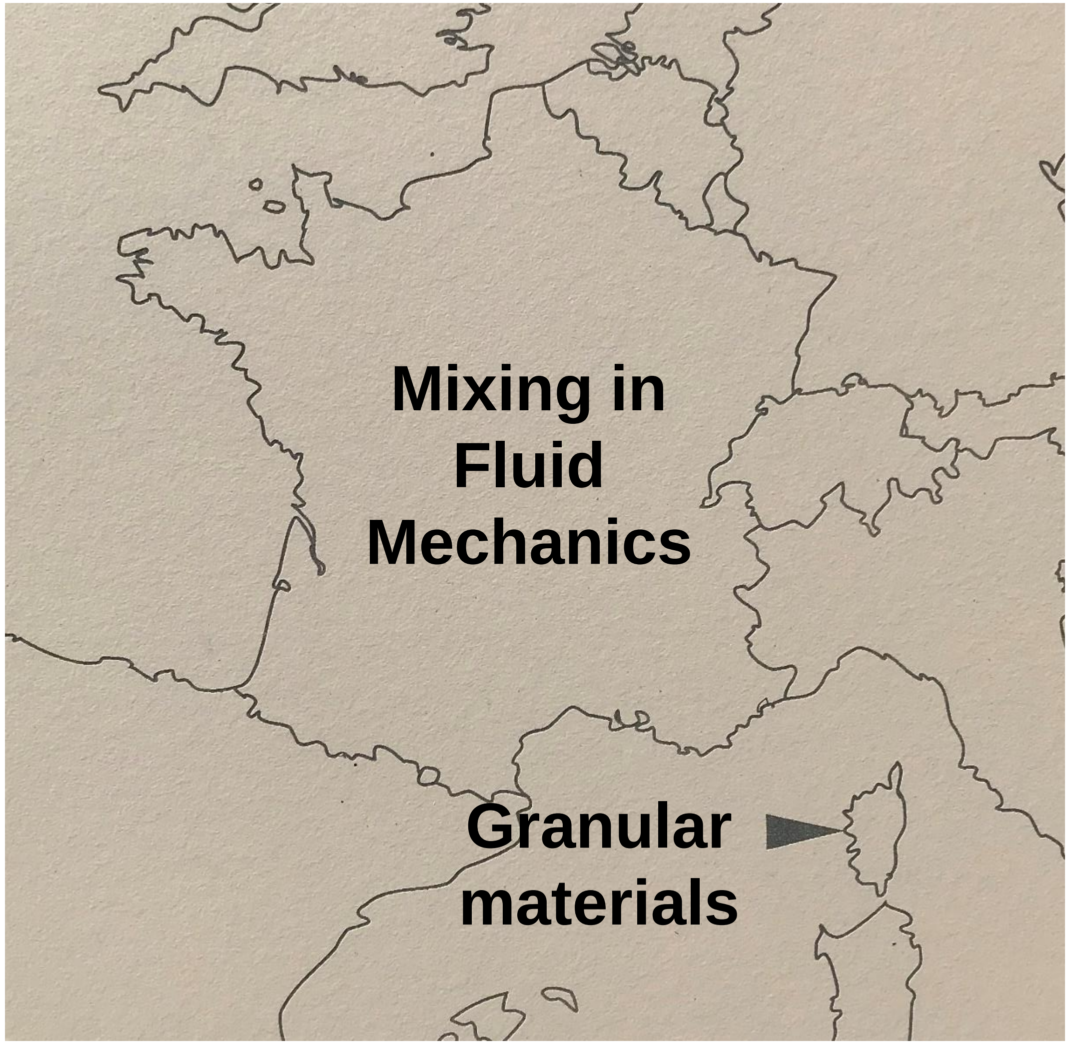
Frontiers of mixing – Cargèse summer school 2023

CoPerMix European ITN project

cnrs advancing the frontiers

Dispersion in granular media

- Granular media
- Rheology
- Diffusion
- Segregation

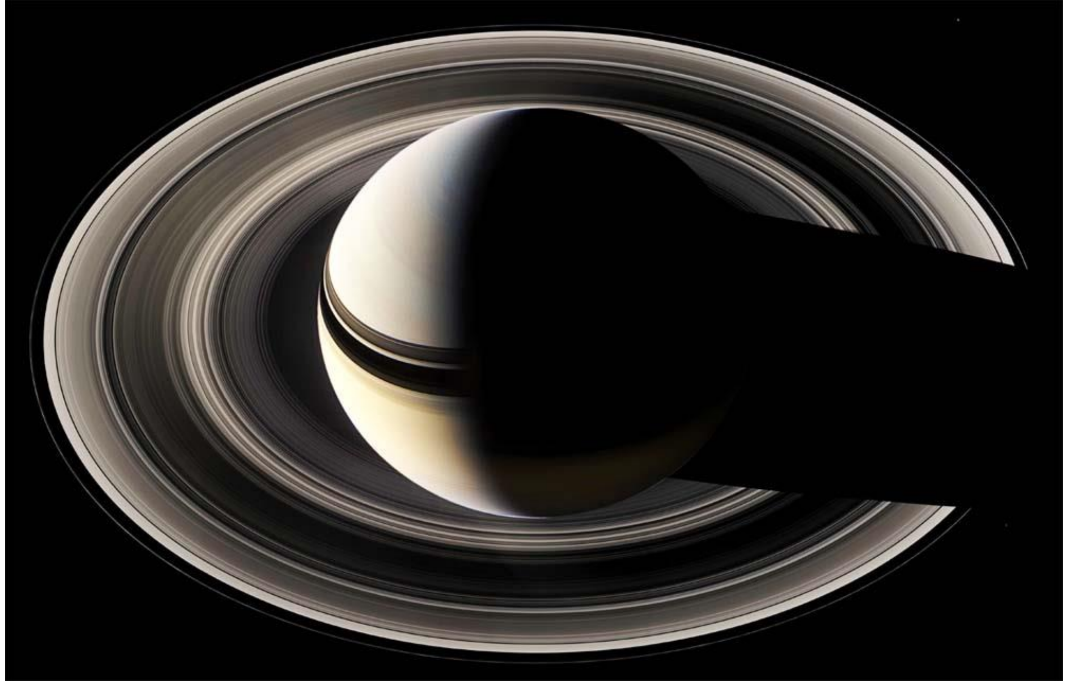


Industrial context

Materials as granular media are the 2nd most frequently used

Food
Mining
Building materials
Chemical engineering
Pharmaceutical

Issues for storage, transport, process



In every day life

Mixing is ubiquitous in industry

- **Food industry**
  - Usually in close batch (sanitary issues)
  - Textural properties, rheology
- **Chemical products**
- **Pharmaceutical**
  - Homogeneity, mostly powders
- **Building materials**

Cohesive grains from sand grains in glass production to powders

Grains/powders
+ liquids

GRANULAR MATERIALS

1 µm — 100 µm →

colloid — powder — granular materials

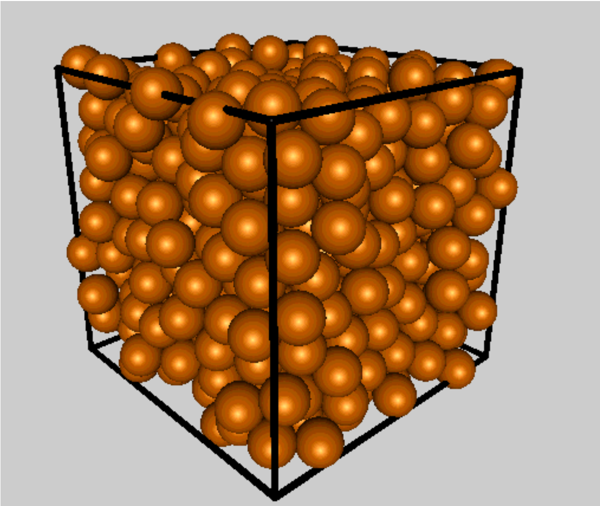
Large particles : no thermal energy

$kT \ll mgd$ **no Brownian motion**

Hard particles : steric exclusion

friction, cohesion, viscous fluids between grains in contact

SOLID FRACTION Φ

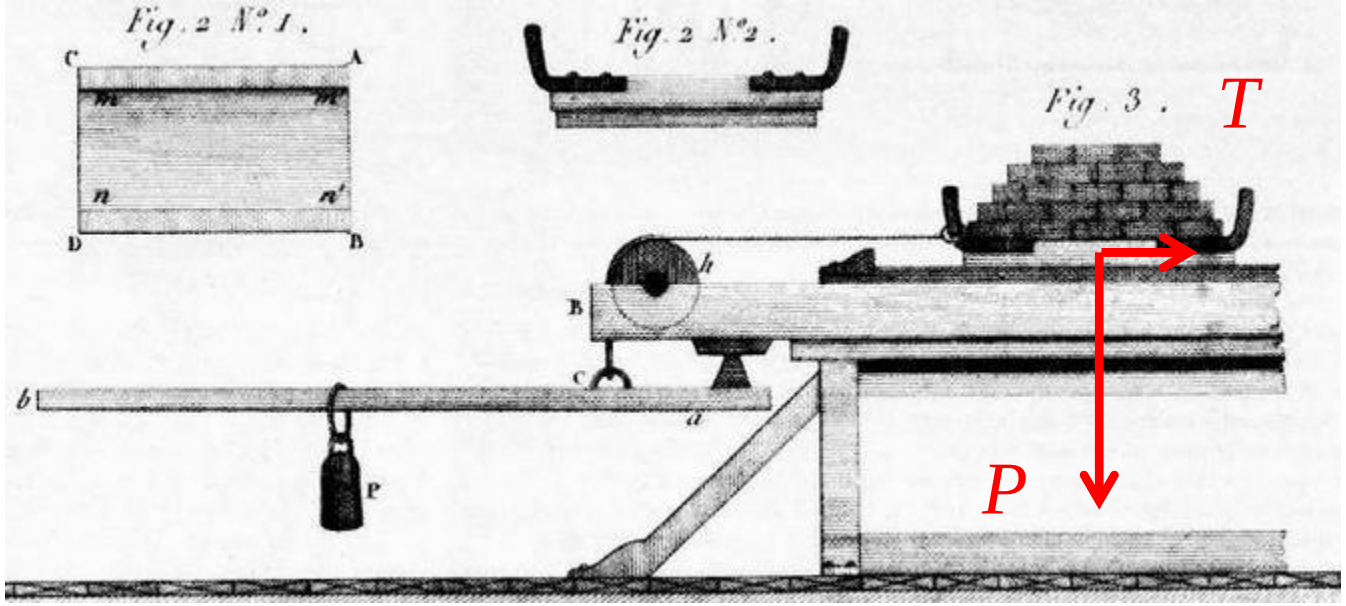


Φ

0,55	0,64	0,74
Random loose packing	**Random close packing RCP**	**Ordered packing**

J. Kepler (1611), T. Hales (2014)

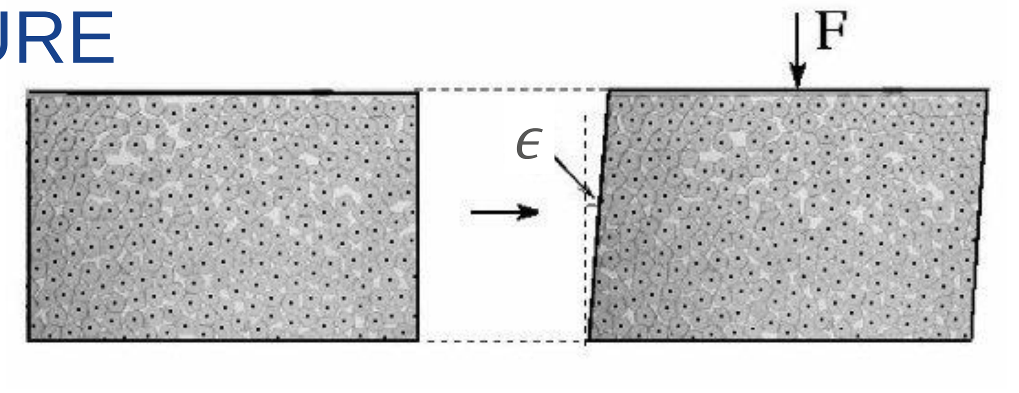
Charles-Augustin Coulomb 1736-1806



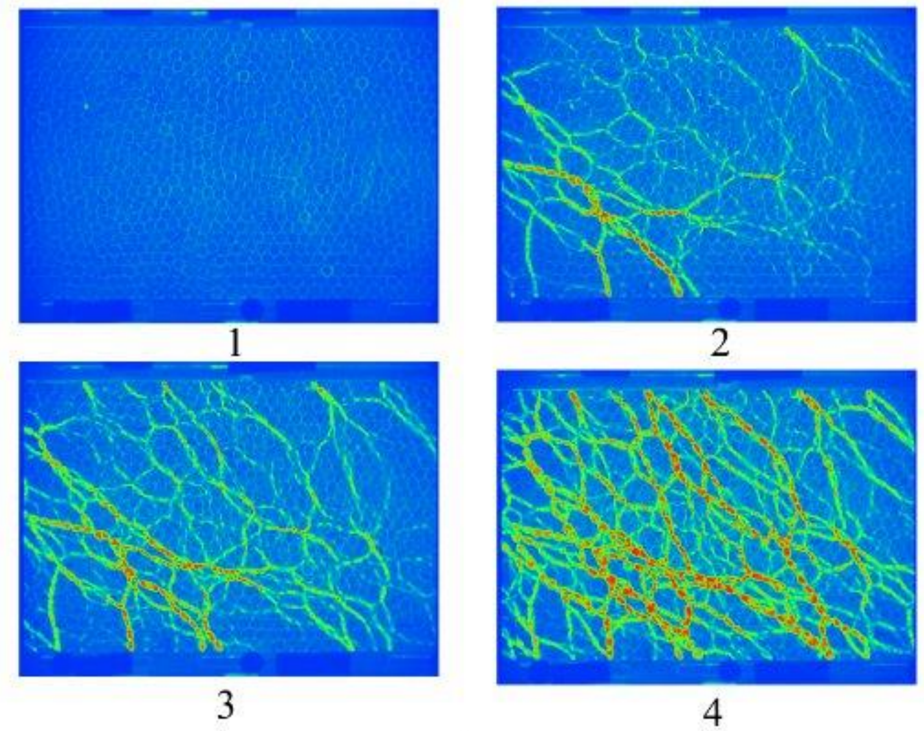
The tangential force T is equal to the normal force P times a friction coefficient μ

$$T = \mu P$$

EXAMPLE—SIMPLE SHEAR CREATES TEXTURE



(a)



(b)

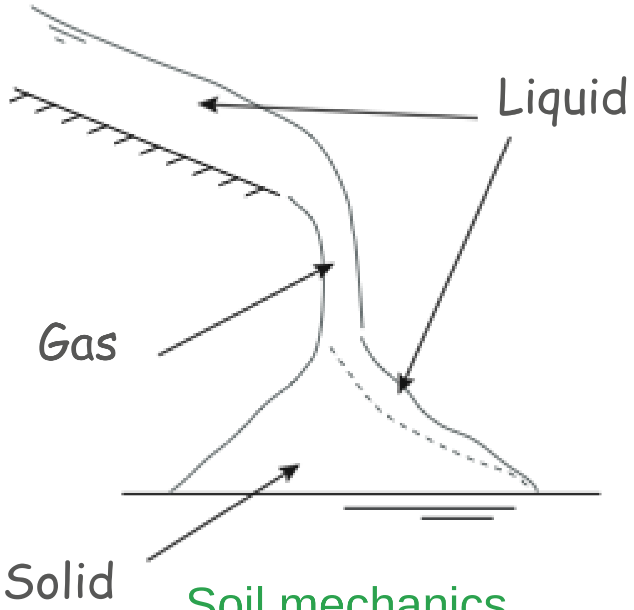
Fabric tensors to describe the anisotropy of contacts orientation, of contact forces, …

Dispersion in granular media

- **Granular media**
- **Rheology**
- **Diffusion**
- **Segregation**

BETWEEN SOLID AND LIQUID

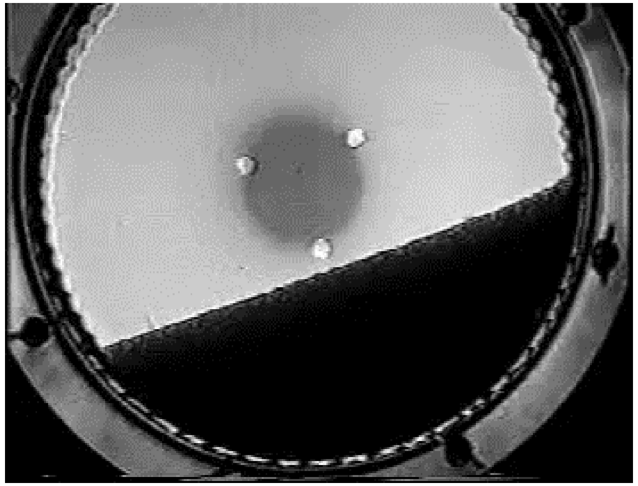
Kinetic theory
(fast, dilute, collisions)



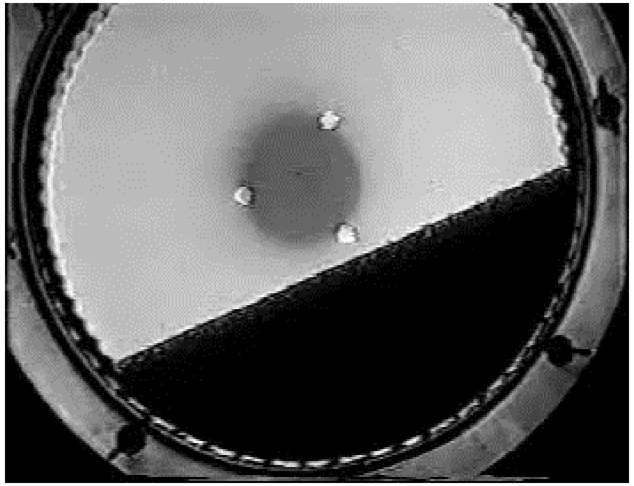
« Fluid mechanics »

Dense, flowing, fast

Soil mechanics
(quasi-static deformation, dense, frictional)

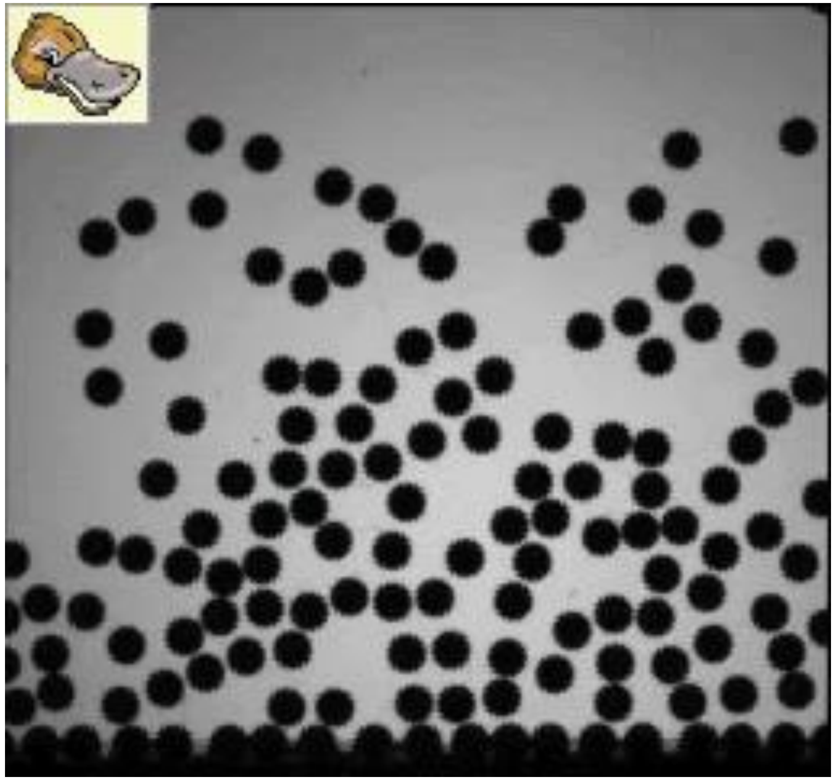


Avalanches

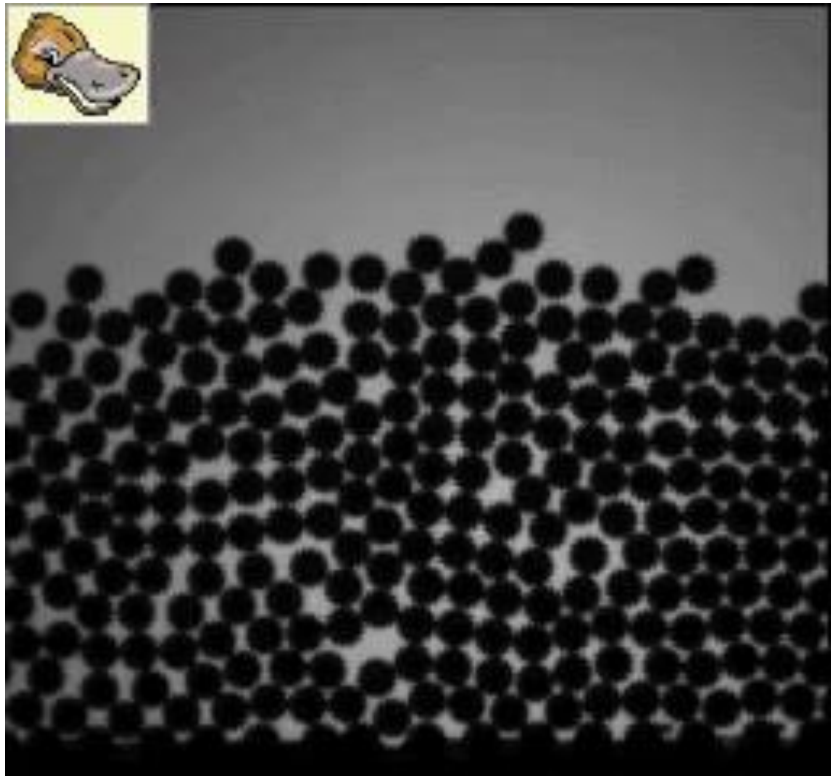


Continuous flow

FLOW REGIMES



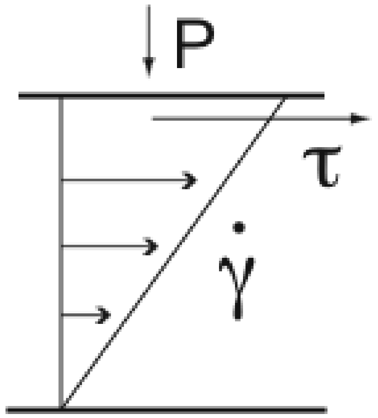
Dilute



Dense

Locale frictional rheology $\mu(I)$

(no elasticity)



ρ_s, d

Model materials :
- disks, spheres : diameter d, mass m
  (mass density ρ_s)
- slight polydispersity

Elastic collision τ_c $\tau_c \ll T_1, T_2$

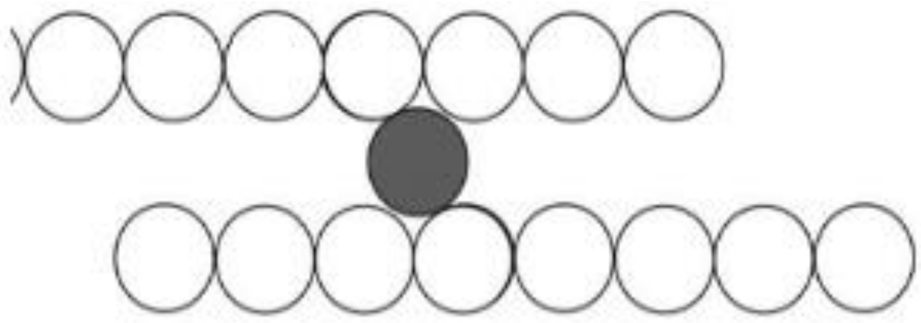
Dimensional analysis

I = T1/T2 :

- rearrangement time $\dfrac{d}{\sqrt{P/\rho_s}}$
- shear time scale $\dot{\gamma}^{-1}$

$$\frac{\tau}{P} = \mu(I)$$ Friction coefficient

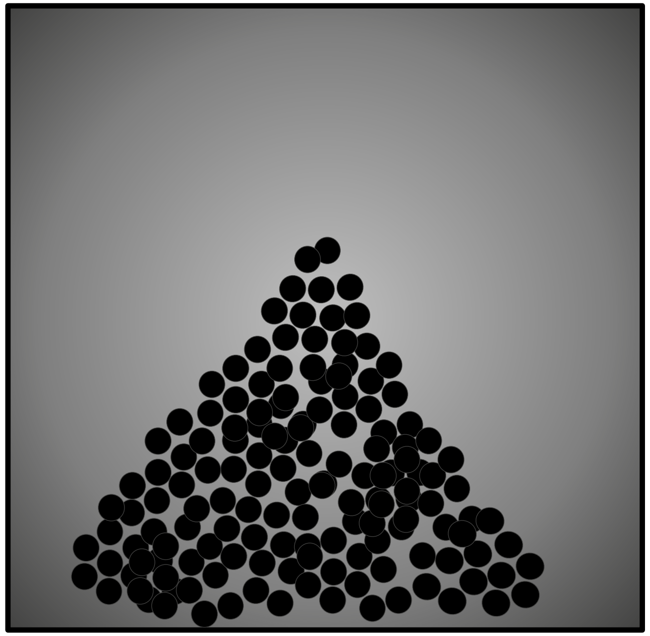
$$I = \frac{\gamma d}{\sqrt{P/\rho_s}}$$ Inertial number



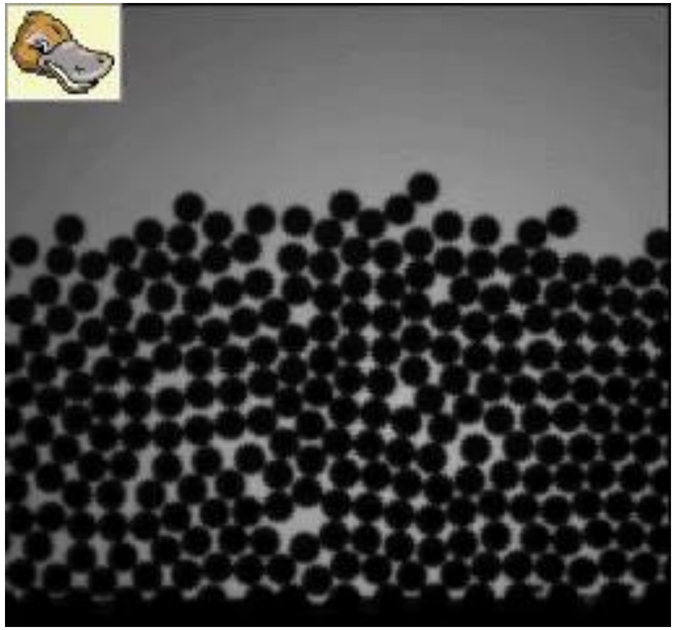
FLOW REGIMES

$$I = \frac{\gamma d}{\sqrt{P/\rho_s}}$$

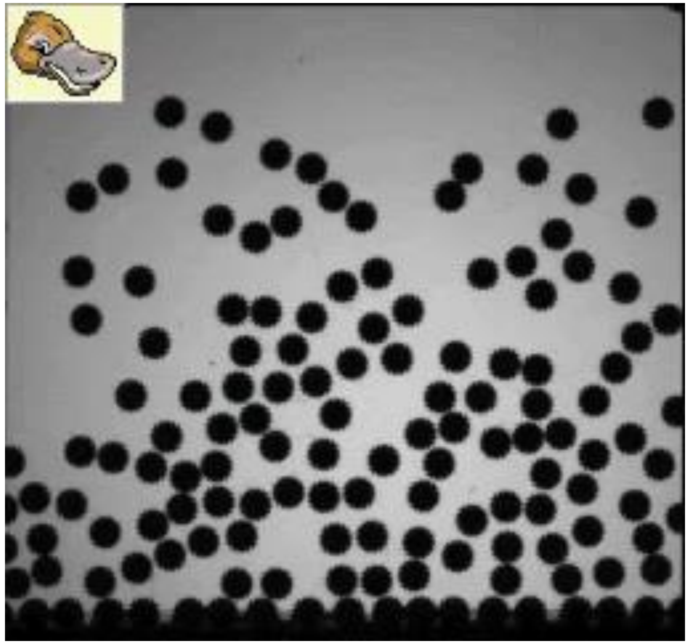
0 0,01 0,1 1



Solid



Liquid



Gas

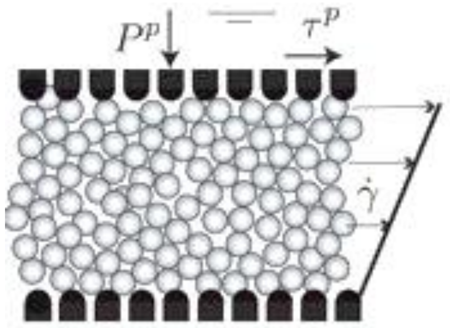
For dense suspensions? (at low Reynolds without inertia)

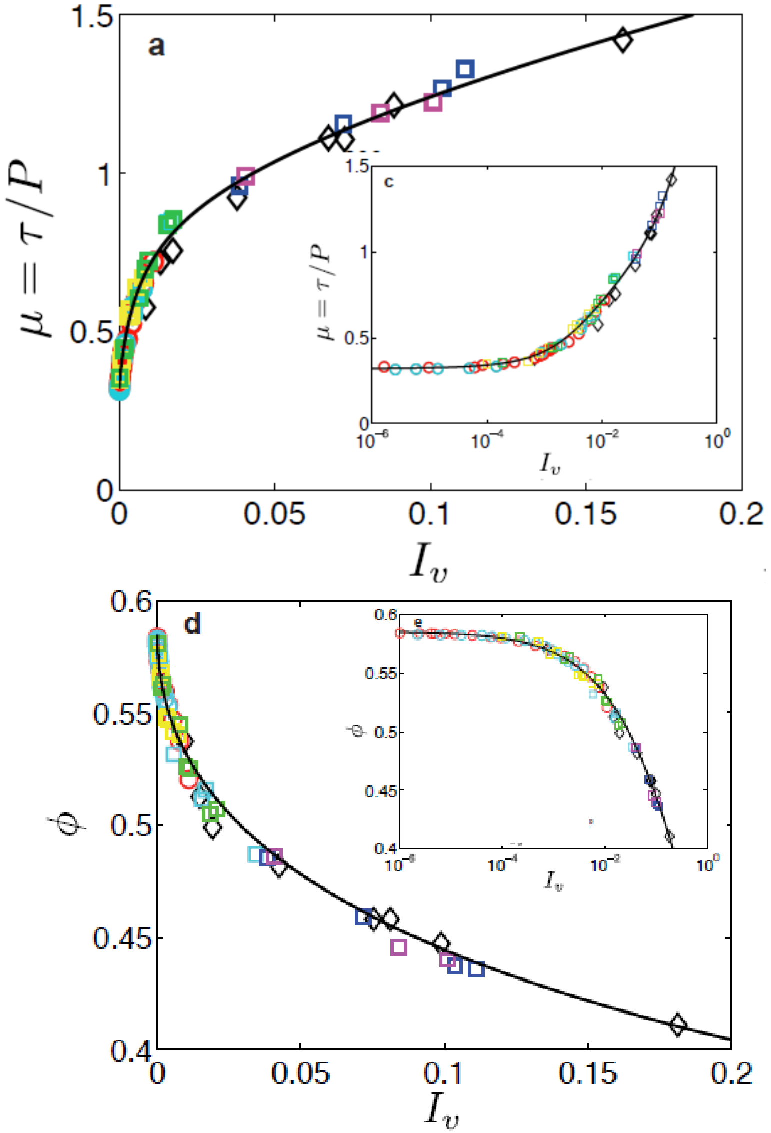
$$t_{micro} = \frac{\eta_f}{P^p}$$

$$I_v = \frac{\eta_f \dot{\gamma}}{P_p}$$

$$\tau = \mu(I_v) P_p$$

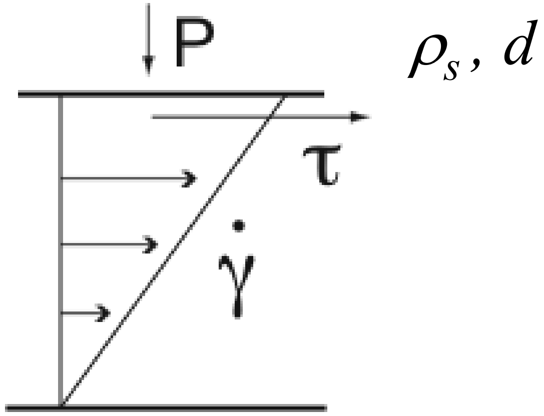
$$\phi = \phi(I_v)$$

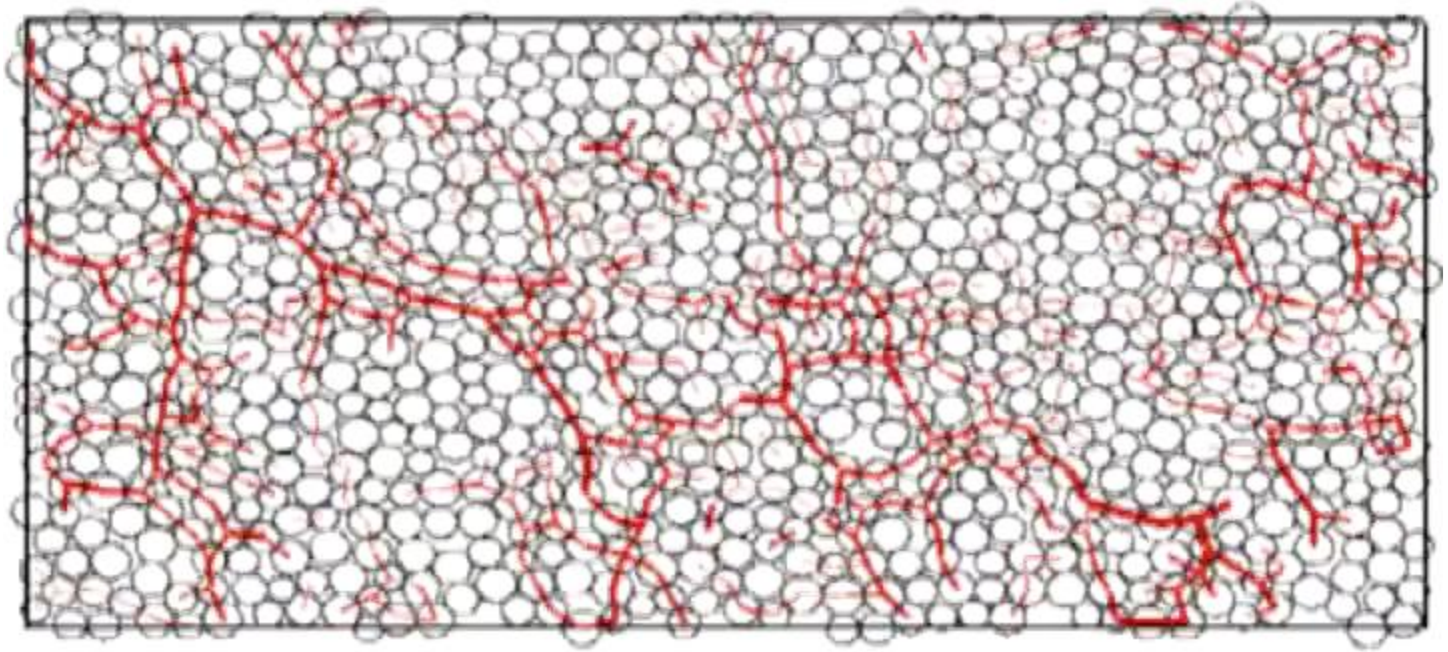




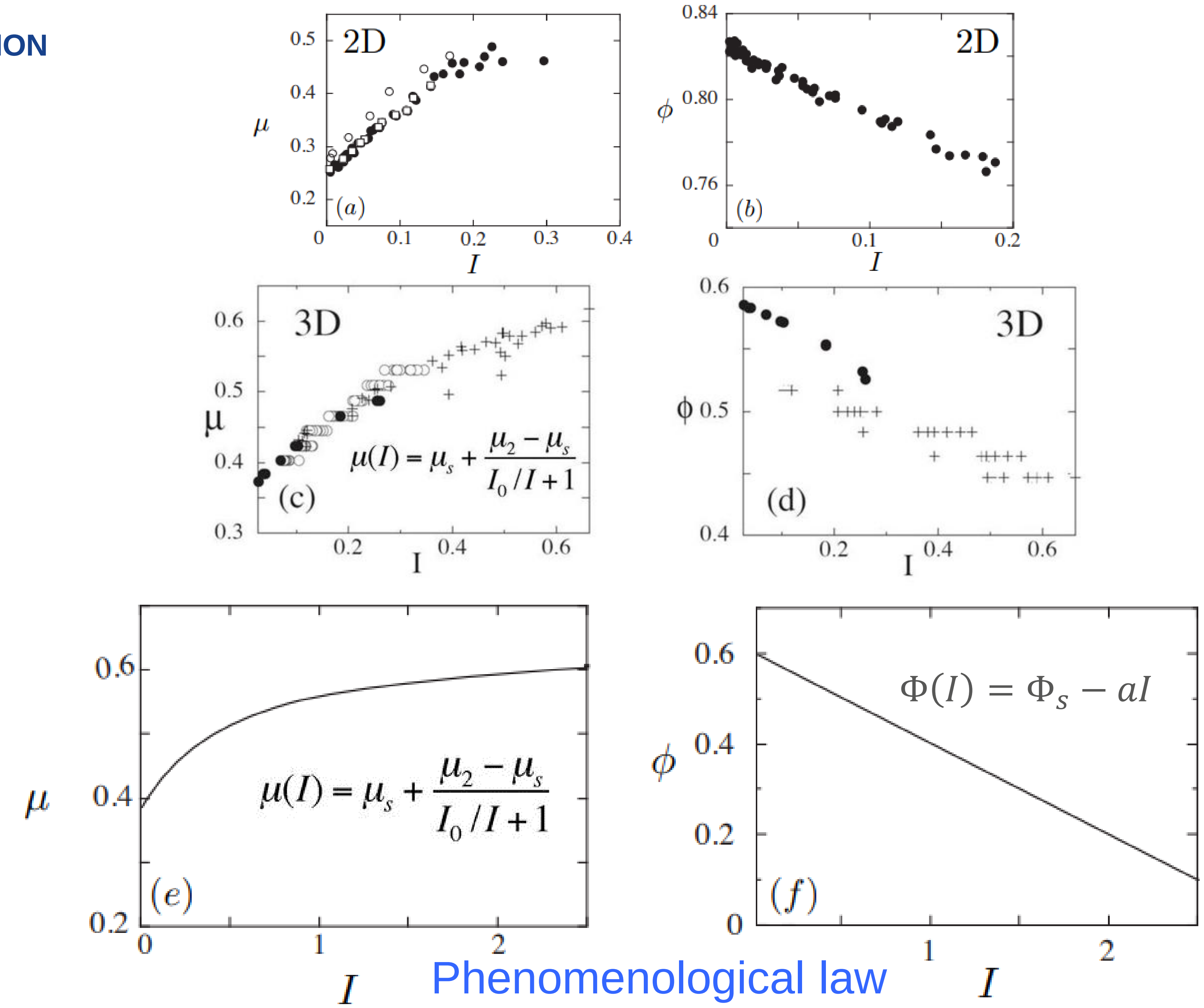
Boyer, PRL 2011

Controlled pressure and shear rate
Discrete numerical simulation



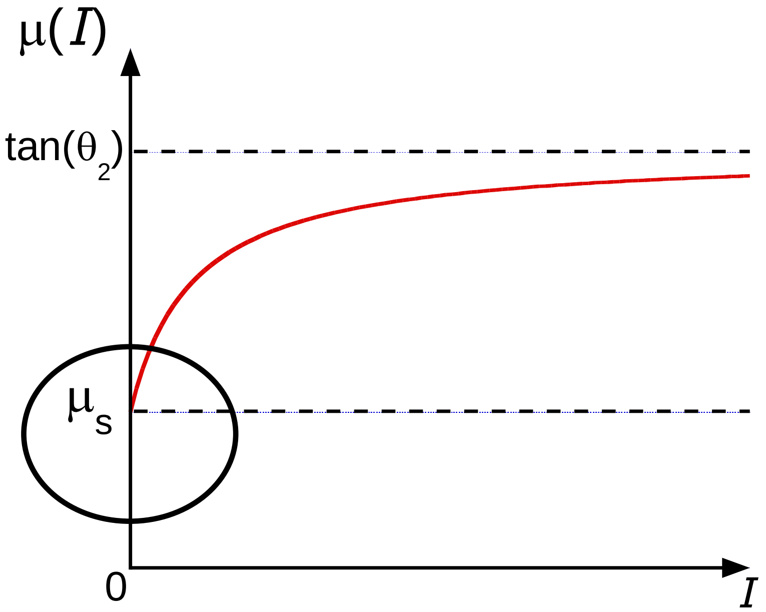


$\mu(I)$ FUNCTION



Phenomenological law

CONSTITUTIVE LAW FOR GRANULAR FLOW: $\mu(I)$ RHEOLOGY



$$I = \frac{\gamma d}{\sqrt{P/\rho_s}}$$

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}$$

Discrete numerical simulations and experiments

eg. for glass beads : μ_s=tan(20.9°), μ_2=tan(32.76°), I_0=0.279

Threshold
No flow $\tau < \mu_s P$

Da Cruz and Chevoir PRE 2003, 2005
Iordanoff and Khonsari ASME Journal of Tribology 2002
GdR MiDi EPJE 2004
Jop JFM 2005

$I \ll 1$

$$\mu(I) \approx \mu_s + bI$$

$$\tau \approx \mu_s P + bd\sqrt{\rho_s P}\dot{\gamma}$$

Viscoplastic material, $\eta \propto P^{1/2}$!

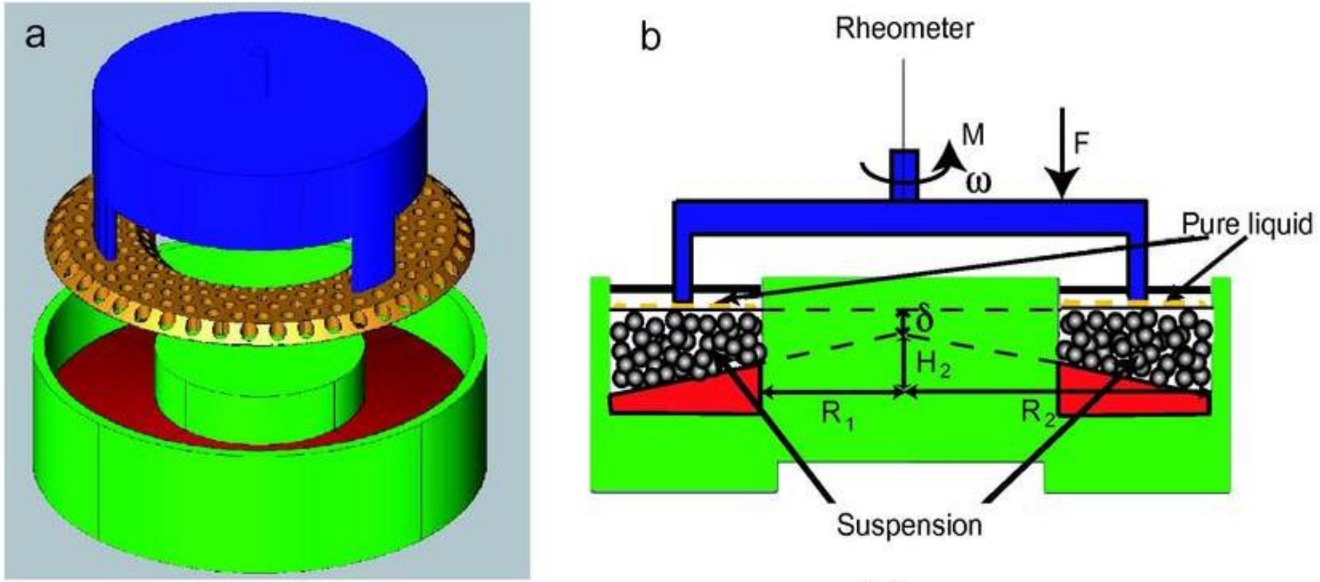
HOW TO MEASURE THIS RHEOLOGY

Shear experiment

Controlled pressure (variation of volume fraction)

$$\Phi = \Phi(I) \qquad \Phi(I) = f_1^{-1}(1/I^2)$$

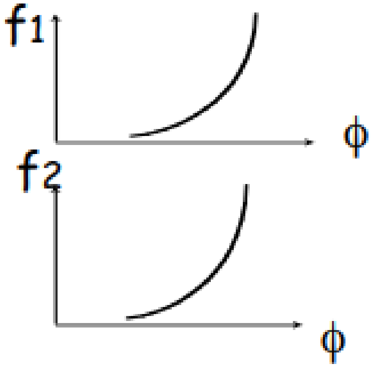
$$\tau = \mu(I)P \qquad \mu(I) = I^2 f_2(f_1^{-1}(1/I^2))$$



Controlled volume (fixed volume fraction)

$$\tau = f_1(\Phi)\rho_s d^2\dot{\gamma}^2$$

$$P = f_2(\Phi)\rho_s d^2\dot{\gamma}^2$$

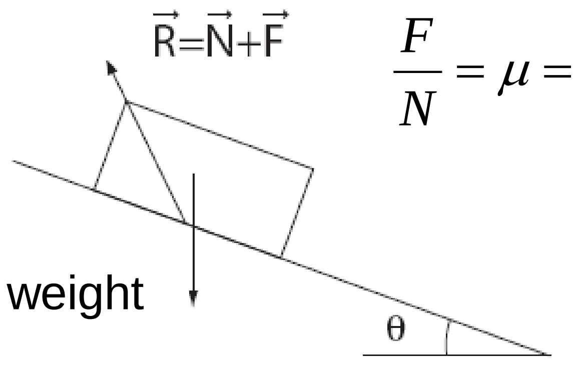


=> Fixed friction coefficient

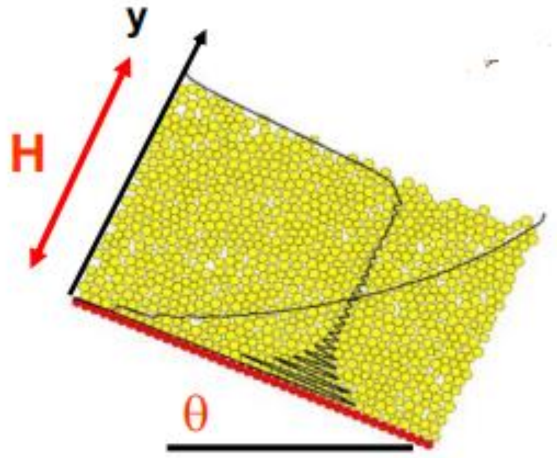
HOW TO MEASURE THIS RHEOLOGY

Steady flows on inclined plane

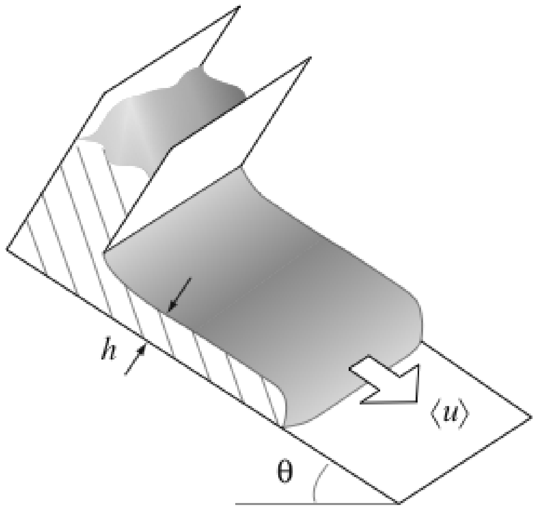
SOLID FRICTION / GRANULAR FLOW



$$\frac{F}{N} = \mu = \tan(\theta_0)$$

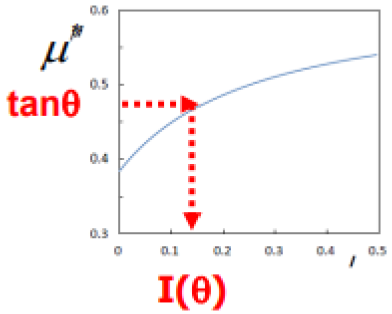


Control parameters:
H et θ



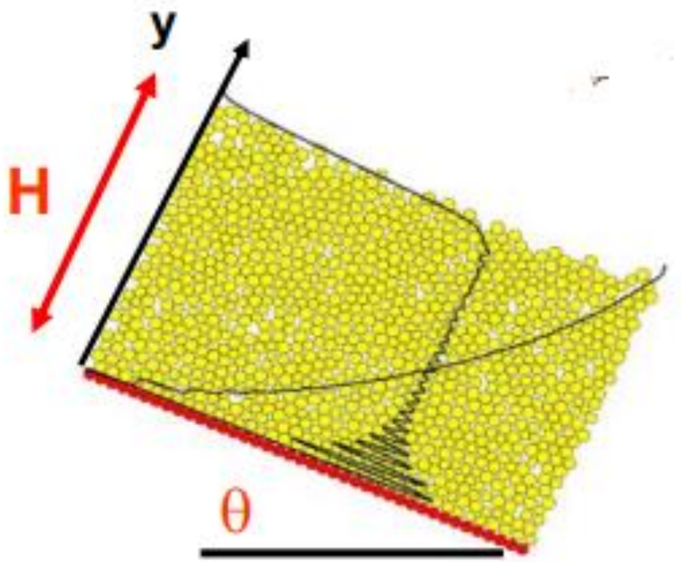
$$\begin{cases} P(y) \approx \rho g \cos\theta (H-y) \\ S(y) \approx \rho g \sin\theta (H-y) \end{cases}$$

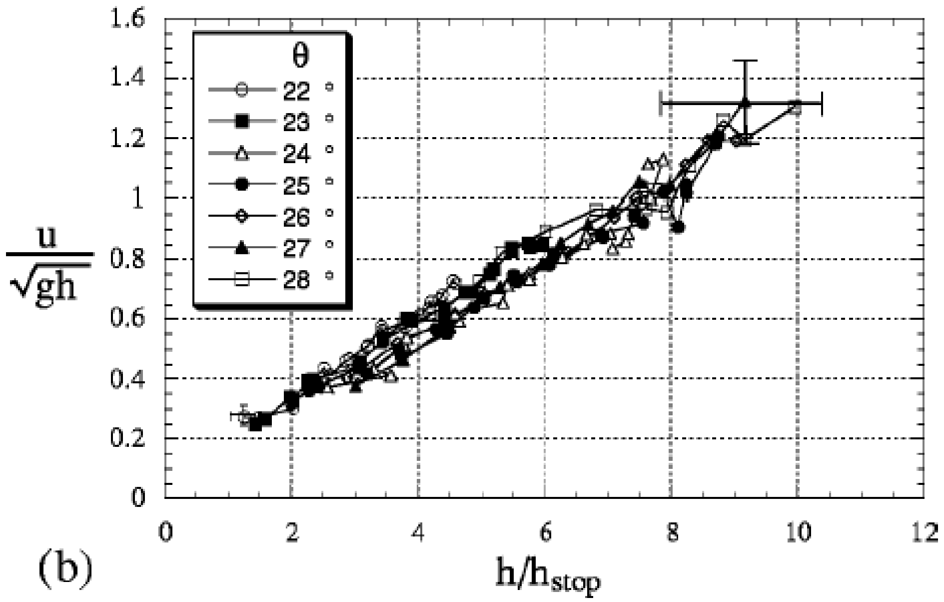
$$\Rightarrow \mu^* = \tan\theta$$



The inclined plane is a rheometer!

INCLINED PLANE FLOWS: VELOCITY PROFILE





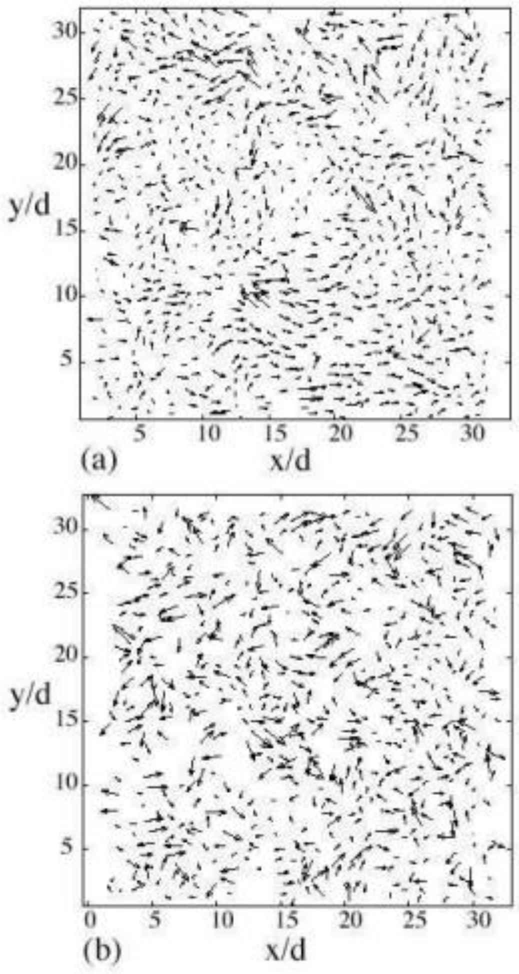
$$I = \frac{\gamma d}{\sqrt{P/\rho_s}}$$

$$\Rightarrow \dot{\gamma}(y) \propto I(\theta)(H-y)^{1/2}$$

$$\Rightarrow u(y) \propto I(\theta)\left[H^{3/2} - (H-y)^{3/2}\right]$$

Bagnod velocity profile (1954)

fluctuations



Pouliquen PRL 2004

2D equation for a visco-plastic fluid

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0,$$

$$\rho_s \phi \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right) = \rho_s \phi g \sin\theta - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},$$

$$\rho_s \phi \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right) = -\rho_s \phi g \cos\theta - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z},$$

$$\tau_{ij} = \frac{\mu(I) P}{\dot{\gamma}} \dot{\gamma}_{ij} \qquad I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

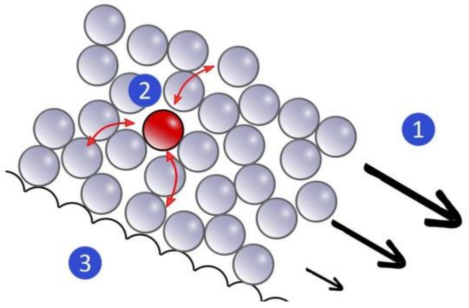
Pressure dependent viscosity

Jop Nature 2006

Limit of the model

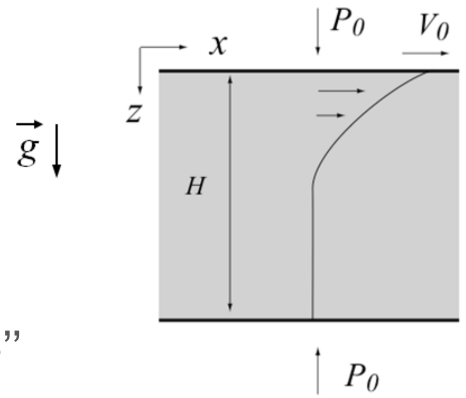
(i) Toward quasistatic flows

Bouzid PRL 2013
Henann PNAS 2013



A mechanical noise characterized by a fluidity *F* (inverse of "viscosity")

Bocquet, PRL 2009



Exponential decrease

Laplacian operator-> propagate the ''noise''

$$l^2 \Delta F = F - F_{bulk}(local)$$

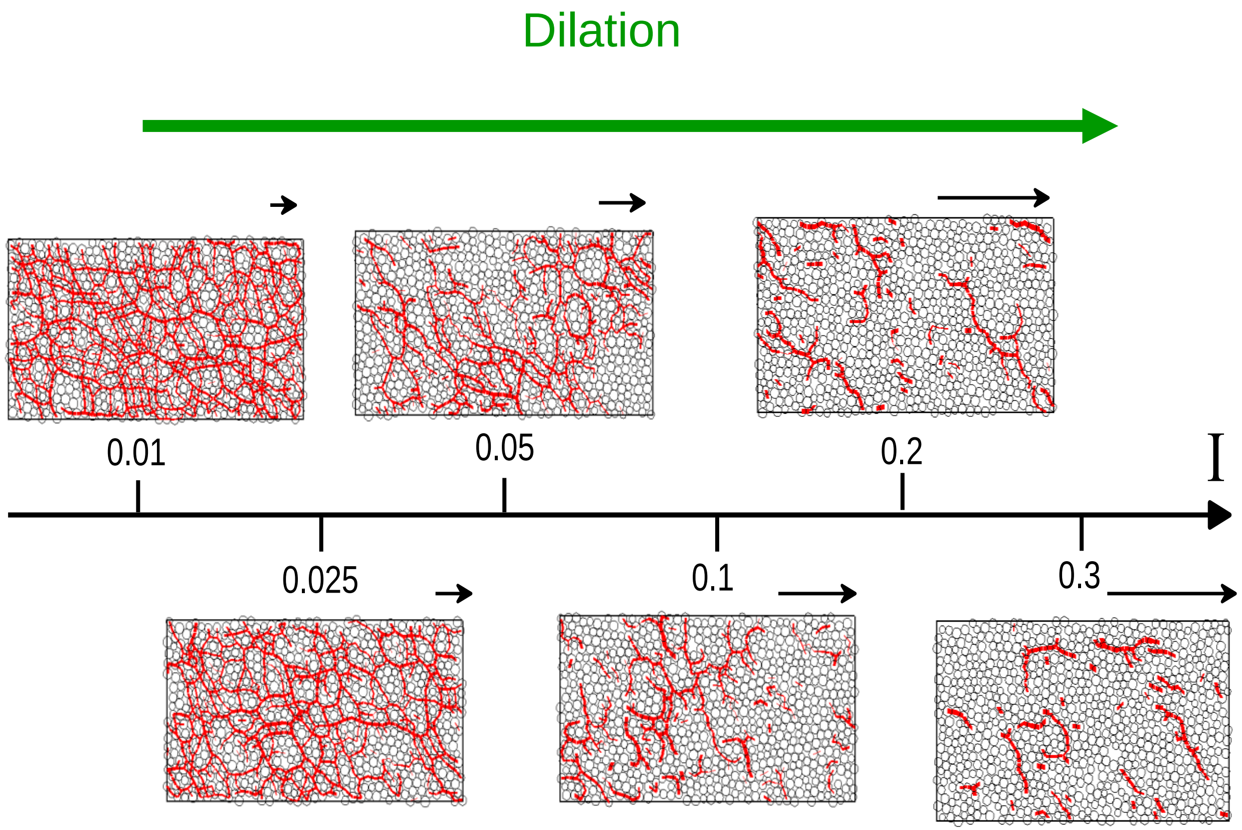
It reproduces finite size effects
(deformed velocity profiles, jamming, …
Important for applications: silos, proppants)

Jop, CR Physics, 2015

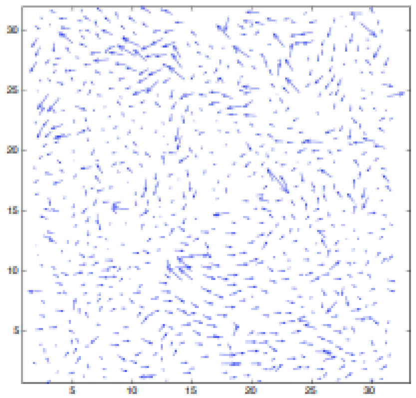
(ii) Toward collisional regime

Large fluctuations of velocities δV
"Granular temperature" T~ δV^2

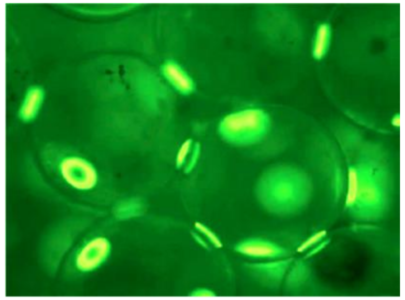
BEYOND THE AVERAGE BEHAVIOUR:
CONTACT FORCES, VOLUME FRACTION, FLUCTUATING DISPLACEMENT



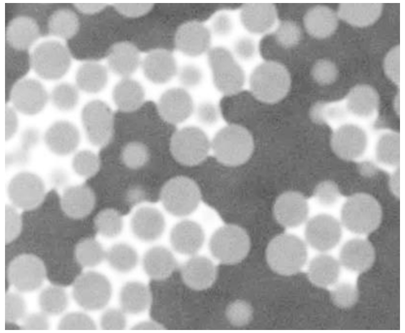
Fluctuations around the mean velocity



Properties of wet granular matter



Herminghaus Adv. Phys. 2005



Tomography Saint-Gobain

pendular

0.032

funicular

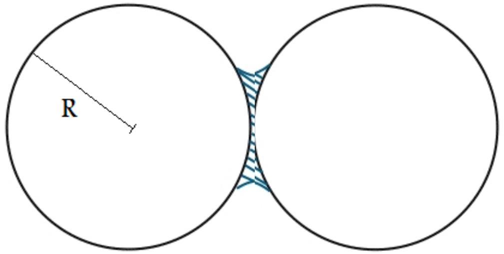
0.12

capillary

$\mathbf{W}=V_{liq}/V_{tot}$

Capillary bridges

- Capillary force :
- Cohesion of the medium



$$F_{cap} = 2\pi R\gamma\cos\theta$$

Capillary bridges coalescence

- Reduction of the cohesion

Percolation of liquid network

- Transition to a dense suspension

Materials	Surface tension (mN/m)
Water	70 mN/m
Water + surfactants	20 mN/m
Molten metal	~ 700 mN/M
Molten oxides	~ 300 mN/m

Fully cohesive granular flows

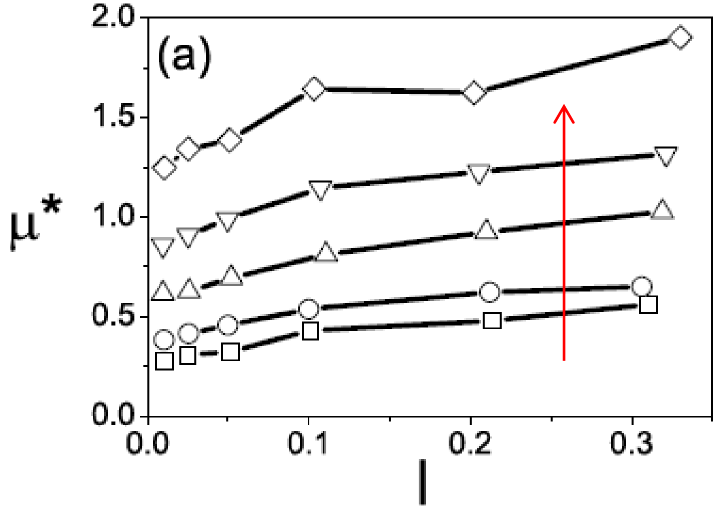
Cohesive number

$$\frac{Nc}{Pd^2} = \eta$$

$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}$$

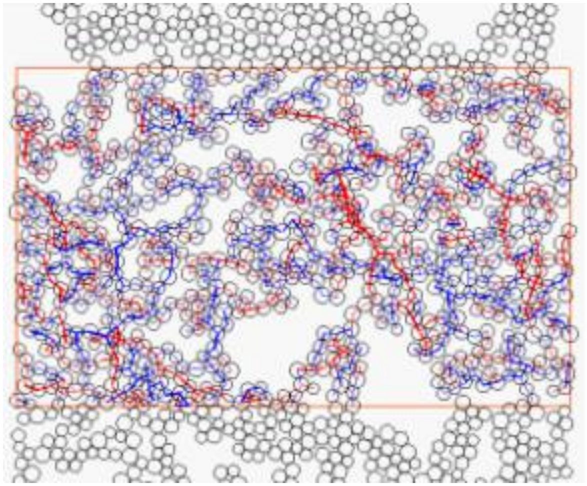
$$\frac{\tau}{P} = \mu(I, \eta)$$

Rognon, EPL 2006

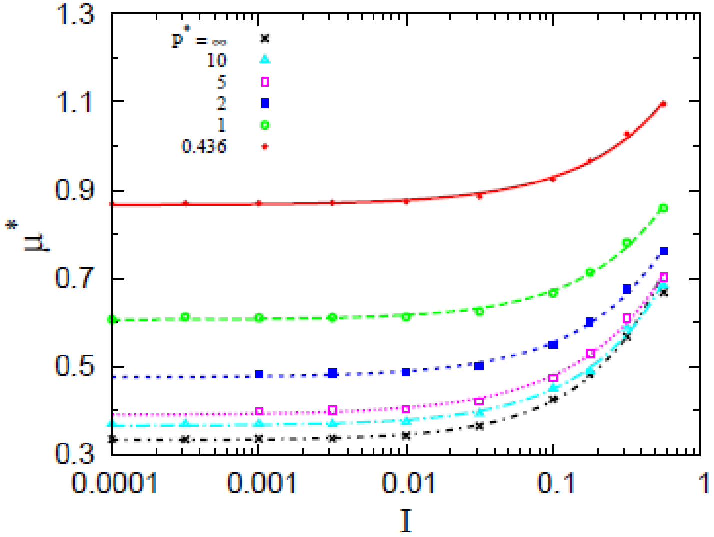


cohesion

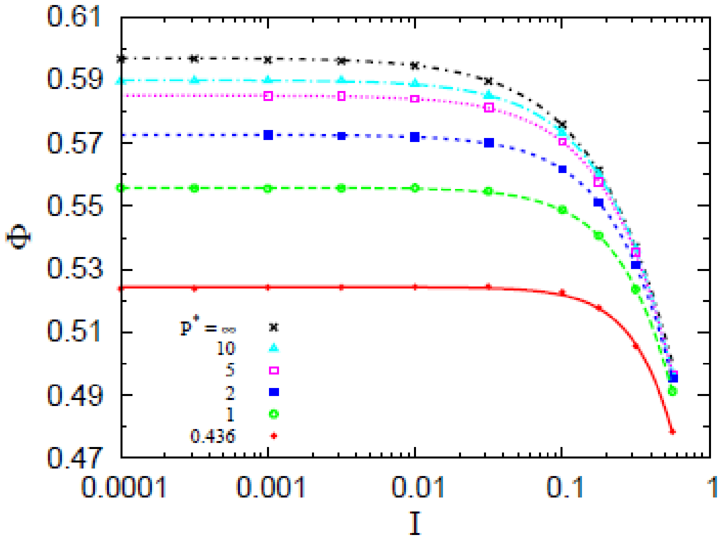
Increasing friction coefficient with the cohesion



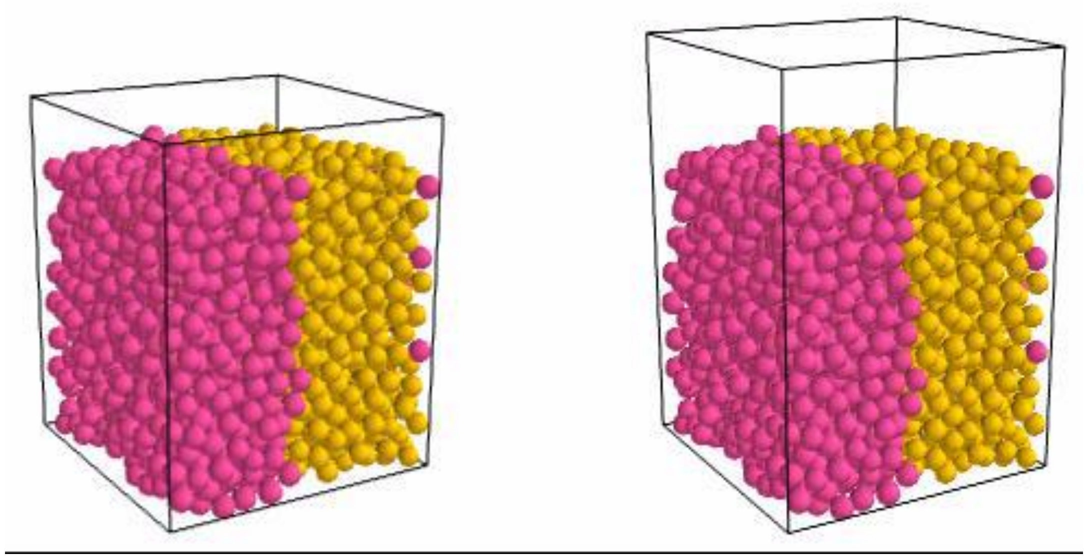
Khamseh 2013, 2015



(b)



MIXING GRAINS AT LOW SHEAR RATE



Dispersion in granular media

- **Granular media**
- **Rheology**
- **Diffusion**
- **Segregation**

Granular diffusion

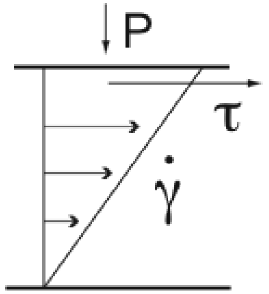
No thermal agitation

Collisions between grains: random direction around the mean velocity.
Length $l = d$, frequency $\nu = \delta v / d$

D= $l^2\ \nu$

Dimensional analysis:
In a simple shear flow

$$D \sim \dot{\gamma} d^2$$



ρ_s , d

$$D \sim d^2 \dot{\gamma} f(\phi)$$ (Savage 1993)

$$D \sim d^2 \dot{\gamma} f(I) \qquad I = \frac{\gamma d}{\sqrt{P/\rho_s}}$$

Granular diffusion

Peclet number

Collisions between grains: random direction around the mean velocity.
Length $l = d$, frequency $\nu = \delta v / d$

$$P_e \sim \frac{UL}{D} \quad \sim \frac{\dot{\gamma} hL}{D} \quad \sim \frac{hL}{d^2}$$

<u>Dimensional analysis</u>:
Flow around an intruder $\quad hL \sim R^2$

Size of shear bands $\quad h \sim 10d$

Size of a cluster $\quad hL \sim s_0^2$

$$P_e \sim 10 - 1000$$

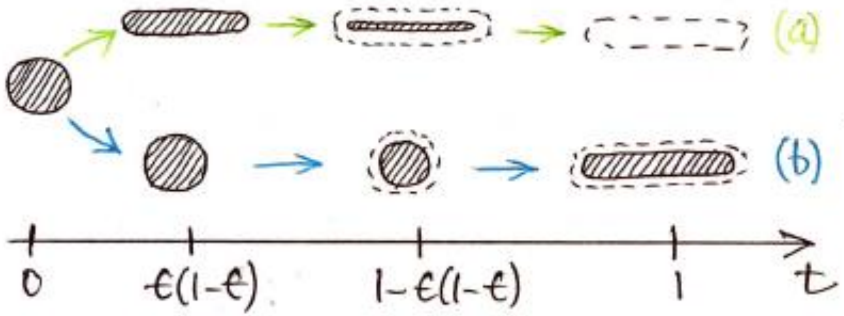
Independancy on $\dot{\gamma}$

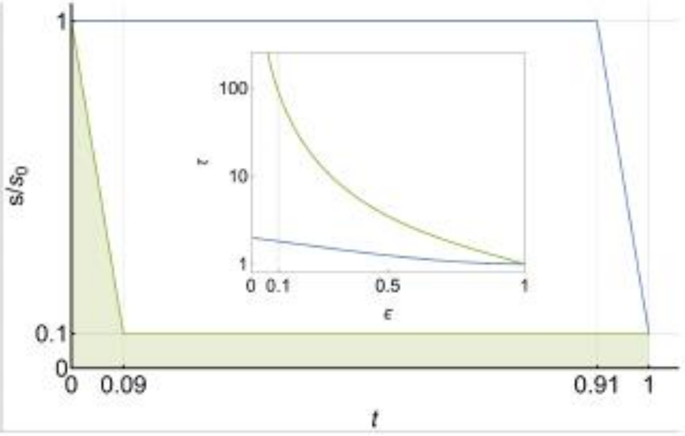
Diffusive profile during advection

$$\sigma_\xi \approx \frac{1}{\sqrt{4\tau}} \qquad \tau = \int_0^1 dt' \frac{D}{s^2(t')}$$

Does history matter?

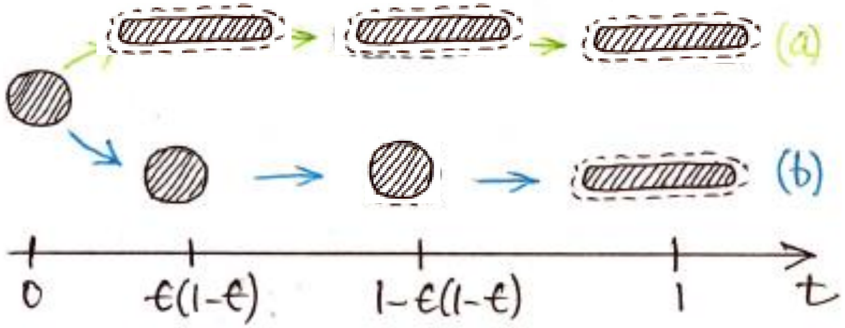
Classical diffusion
D = cst





Villermaux, Ann. Rev. 2019

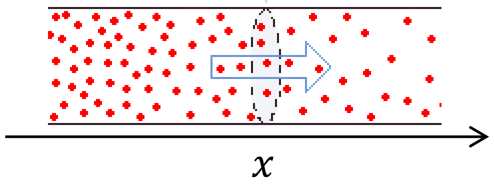
Granular diffusion



NOT in Villermaux, Ann. Rev. 2019

Equation of diffusion

Fick's law for the diffusive flux J



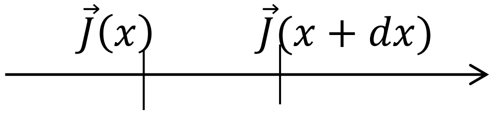
$$\vec{J} = -D\vec{\nabla}C$$

the flux is proportional to the gradient of concentration of the solute

For 1D $$j = -D\frac{\partial c}{\partial x}$$

D is the diffusion coefficient (m^2/s)

Conservation of particles



$$\frac{\partial C}{\partial t} = -\vec{\nabla}\,\vec{J}$$

$$\frac{\partial C}{\partial t} = D\Delta C$$

Diffusion equation

Only if D is constant over the space

When D depends on space:

$$\frac{\partial C}{\partial t} = \frac{\partial D}{\partial x}\frac{\partial C}{\partial x} + D\Delta C$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \frac{\partial D}{\partial x}\frac{\partial C}{\partial x} + D\Delta C$$

1D Advection - Diffusion equation

Origin of the granular diffusion:

Diffusive motion at large time:

Diffusion coefficient increases at low inertial number

$$D = A\dot{\gamma}d^2$$

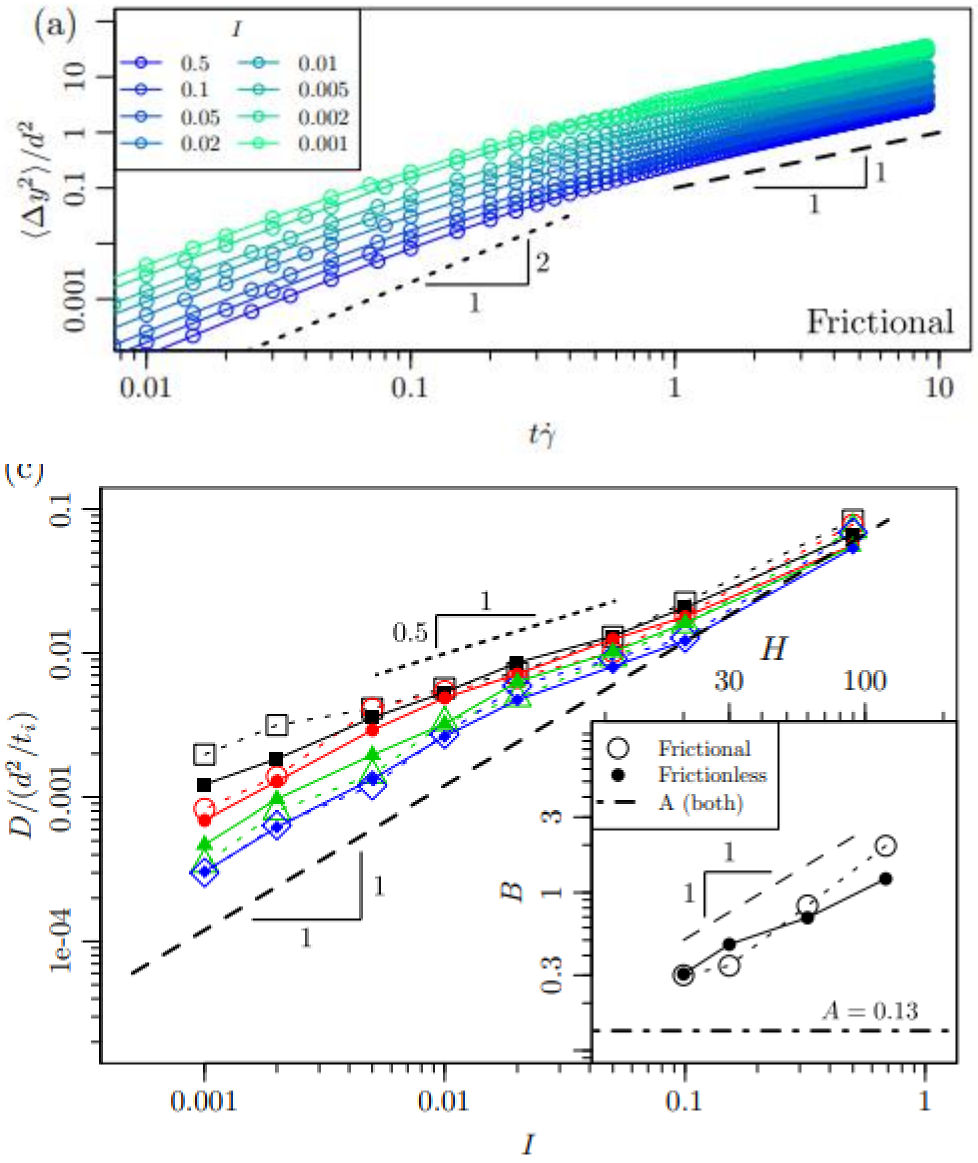
$$\frac{D}{\frac{d^2}{t_p}} = A\dot{\gamma}t_p = AI$$

OK at low and high Inertial number

A~0.1 for large I

A~1 for low I

Different scaling in intermediate inertial number

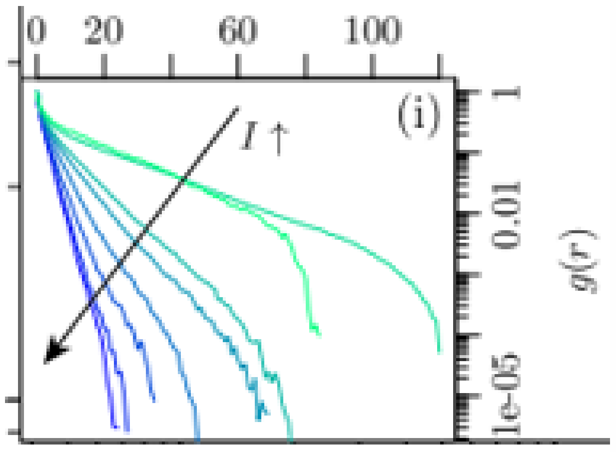


(Kharel PRL 2017)

Correlated movements

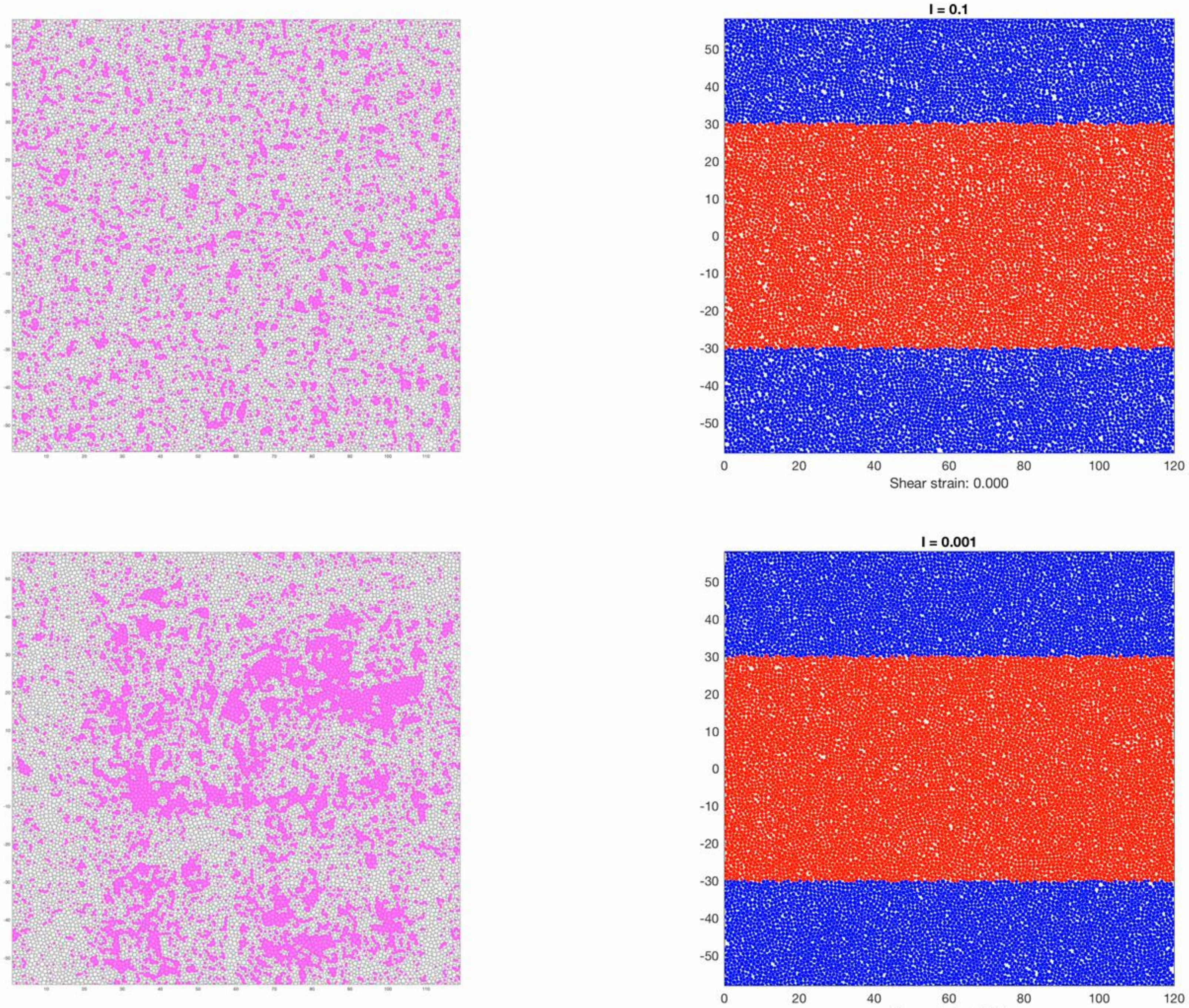
Clusters defined on correlated movements:

g(r) correlation function: Probability that a grain belongs to the same cluster



$$\ell = \frac{\sum_r (r+d)g(r)}{\sum_r g(r)}$$

(Kharel PRL 2017)



Granular vortices (left) and Mixing (right) in Plane Shear Flow of "Frictional" Grains

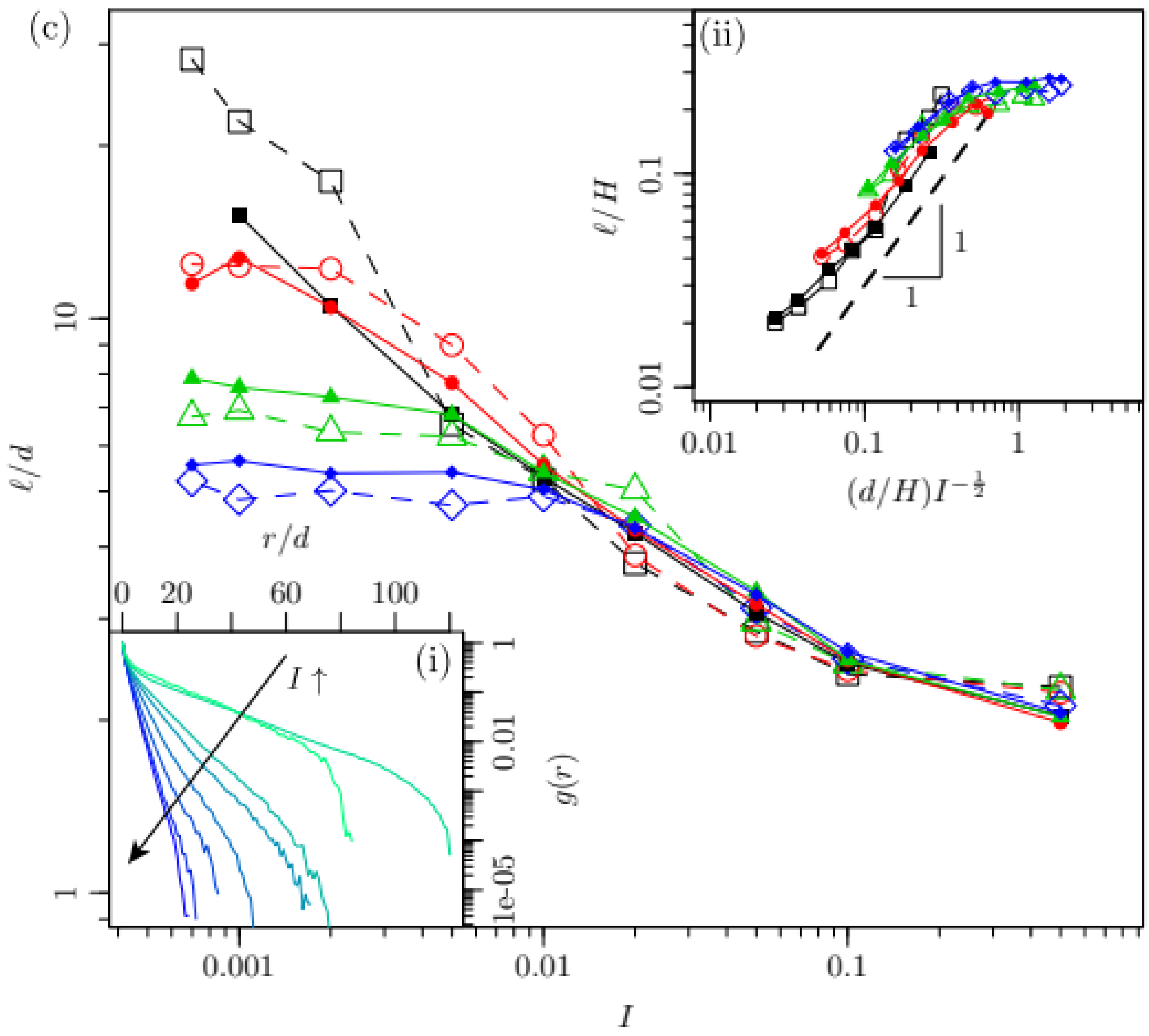
Origin of the granular diffusion:

Correlated movements

Clusters defined on correlated movements:

Scaling of the length at intermediate I

$$\frac{l}{d} \propto \frac{1}{\sqrt{I}}$$



(Kharel PRL 2017)

Origin of the granular diffusion:

Clusters defined on correlated movements:

Simulation facts:

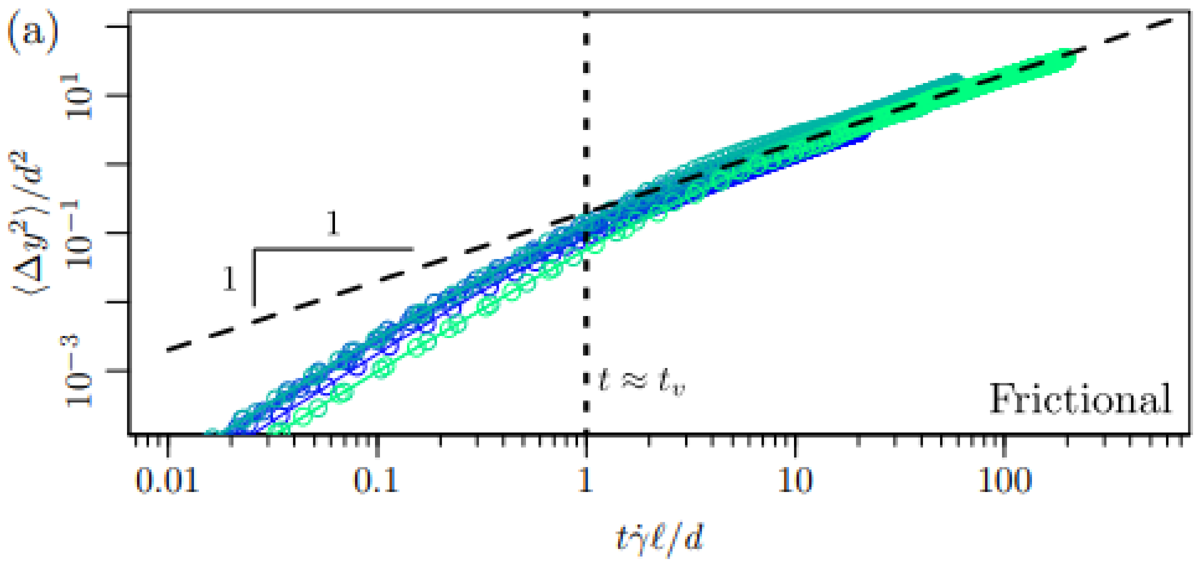
Length   Vortex life time

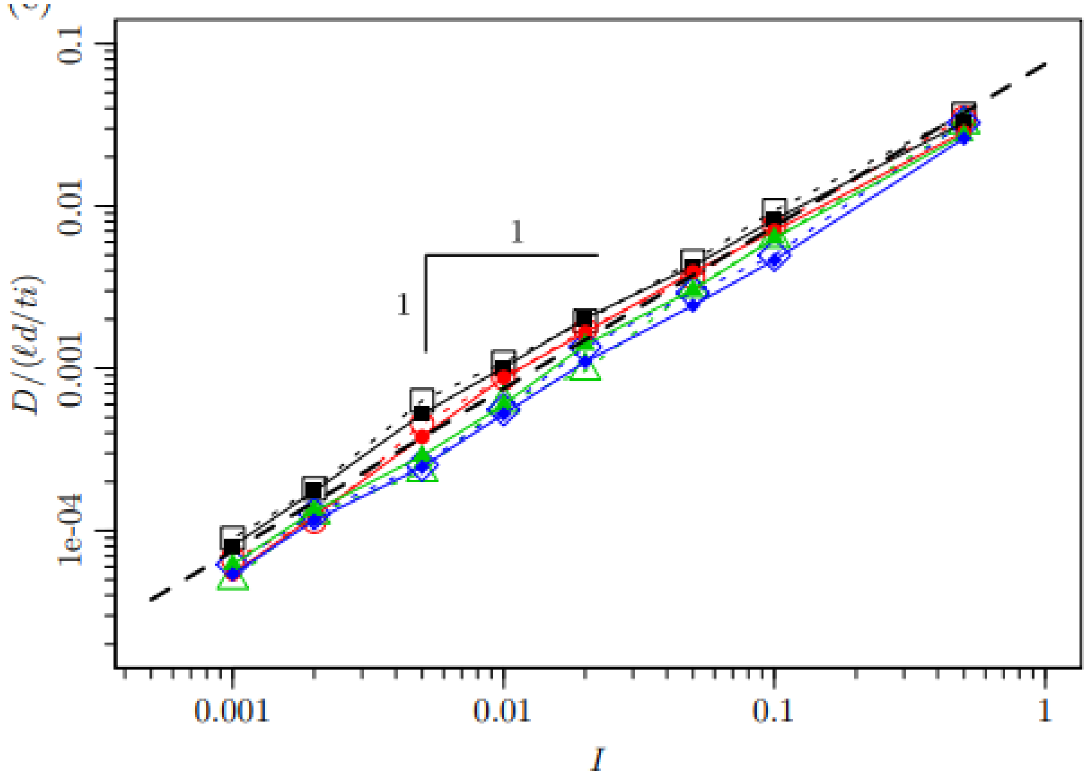
$$\frac{l}{d} \propto \frac{1}{\sqrt{I}} \qquad t_v \propto \frac{d}{\dot{\gamma}\ell}$$

$$D \propto \frac{d^2}{t_v}.$$

$$D \sim \frac{d^2\dot{\gamma}}{\sqrt{I}}$$

(Kharel PRL 2017)





Origin of the granular diffusion:

Correlated movements

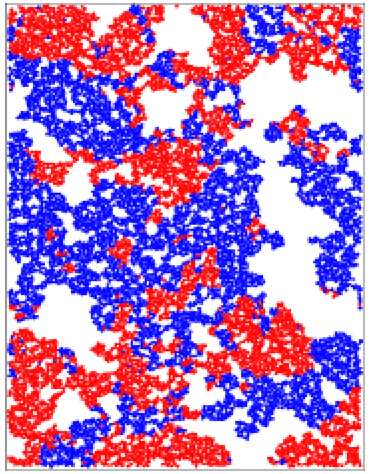
<u>Cohesive case</u>

Clusters Diam.

$$\frac{l}{d} \propto \frac{a + bCo}{\sqrt{I}}$$

Correlation time

$$\tau_{co} \propto (c + kCo) I^{1/2} / \dot{\gamma}$$



(Macaulay JFM 2019)

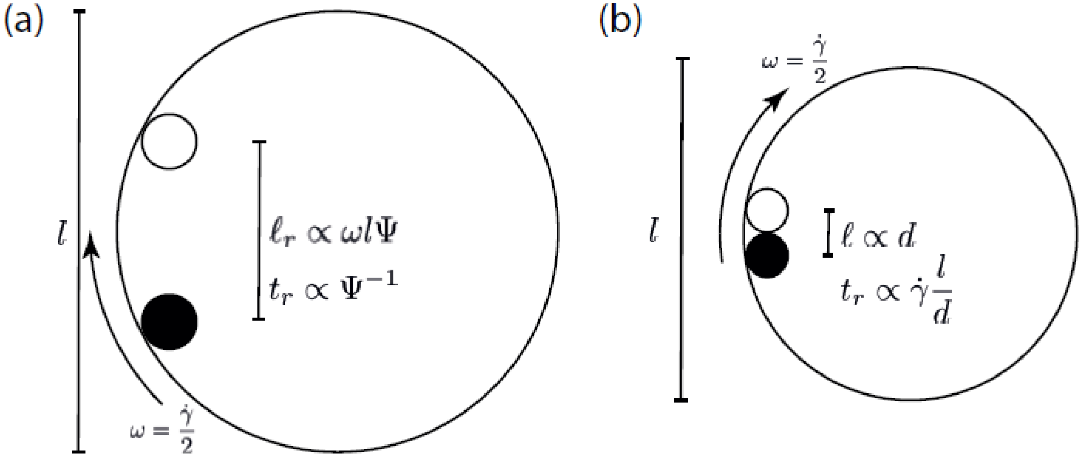
Random step length

$$l_c \dot{\gamma} \tau_{co}$$

Frequency

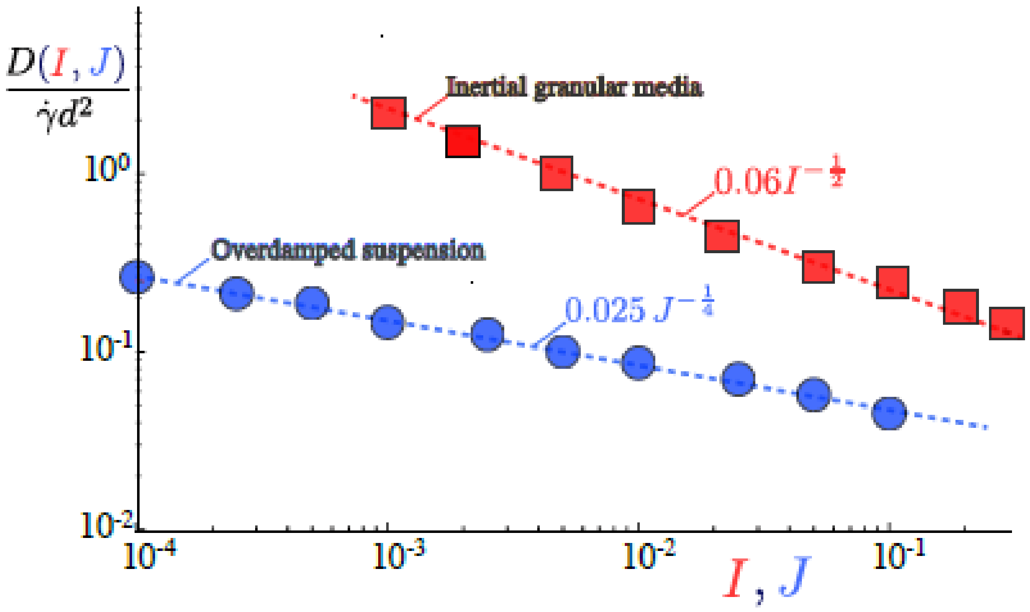
$$1/\tau_{co}$$

$$D(Co) = \frac{(l_c \dot{\gamma} \tau_{co})^2}{\tau_{co}} = f(Co) \frac{d^2 \dot{\gamma}}{\sqrt{I}}$$



Origin of the granular diffusion:

in suspension?



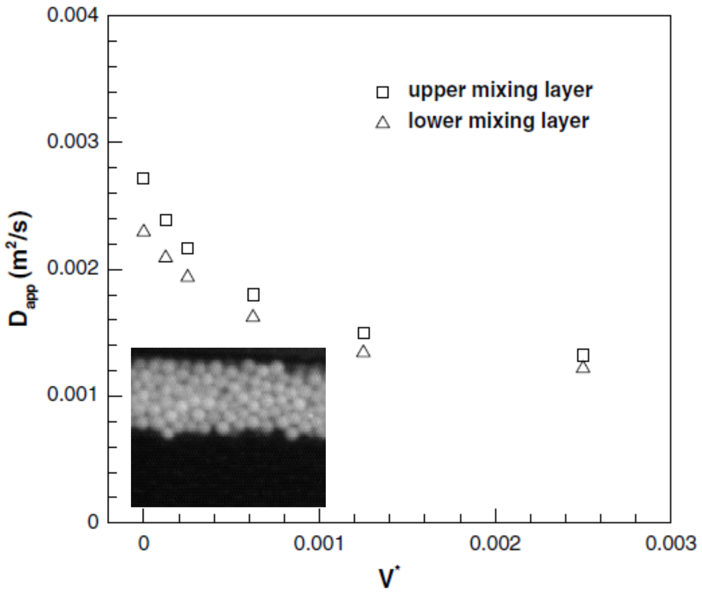
courtesy of Bloen

Seems to governed finally by the volume fraction

Granular diffusion with constant *D*

Cohesive grains

The diffusion coefficient decreases with the volume of added liquid.



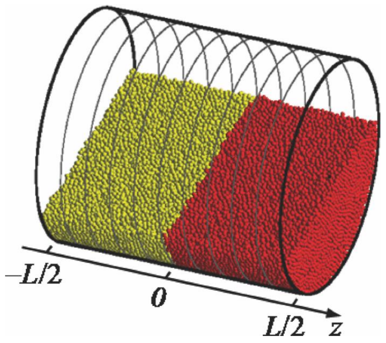
(Hsiau Int. J Mult. Flow 2008)

Bond Number.
Capillary force to the weight of the particle

$$Bo = \frac{\pi \sigma d \cos\alpha}{\frac{1}{6}\pi g d^3 \rho}$$

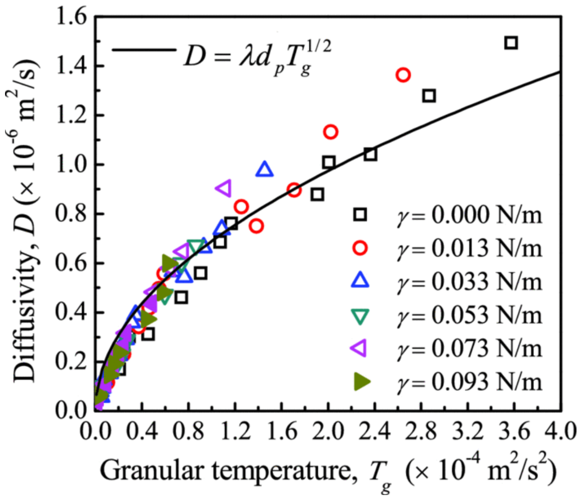
Cohesion Number
Capillary force to the pressure

$$Co = \frac{\pi \sigma d \cos\alpha}{\pi d^2 P}$$



The fluctuations decreases with the volume of the liquid: « longer spring », and viscous dissipation

$$T_g = \frac{1}{3}\langle |\mathbf{v} - \langle \mathbf{v} \rangle|^2 \rangle$$



(Liu Phys. Fluids 2013)

Take home messages

- **Inertial number governes the flow of granular media**
- **Diffusion of the grains is related to the shear rate at first order**
- **The precise history of the streching should not be critical.**

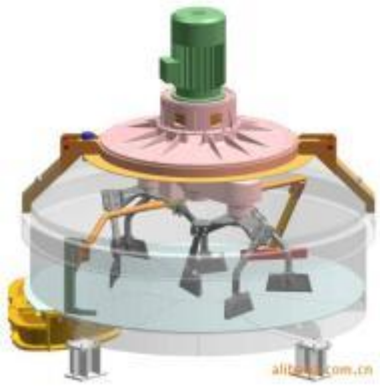
Dispersion in granular media

- **Granular media**
- **Rheology**
- **Diffusion**
  - **Diffusion**
  - **Some flows**
- **Segregation**

Type of industrial mechanical mixers

- **Passive mixers :** e.g. kenics mixer
- **Active mixers :**
  - **Double shafts**
  - **Planetary movement**
    - Spatial homogeneity: scrape the borders

Guntert & Zimmerman TSM Series Twin Shaft Compulsory Mixing System

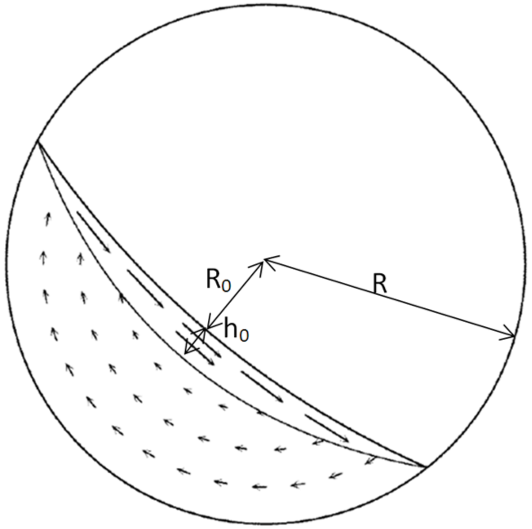


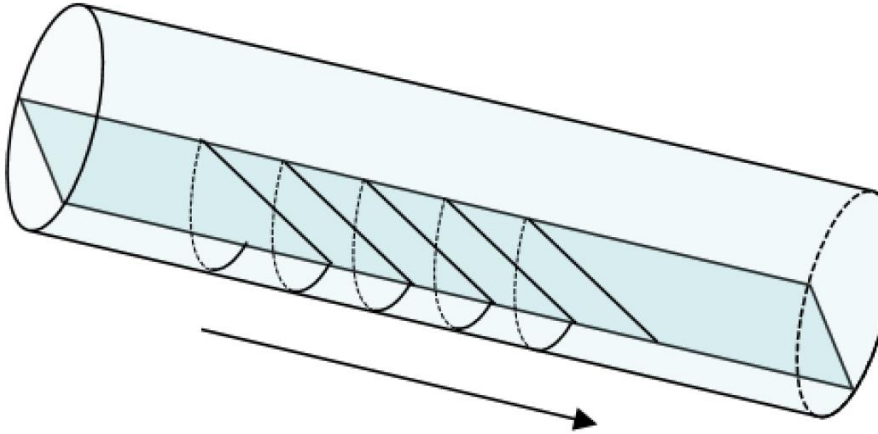
  - **Impellers**

  - **Specific for grains**

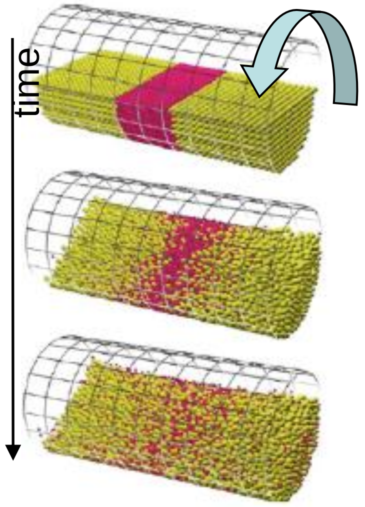
Curing materials in open flows

- **Grains and powders**
  - Rotating kiln
  - Powders
  - Hot temperature
  - Sintering





Axial dispersion in the rotating drum



Three origins of the dispersion of the grains

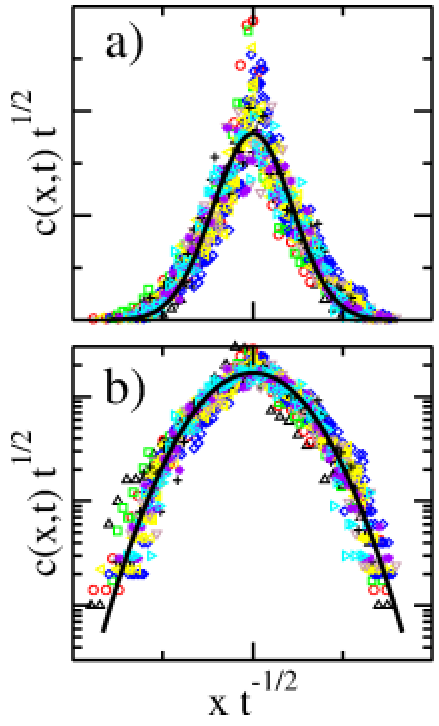
- the intrinsic horizontal diffusion,
- a geometrical effect linked to the shape of the flowing layer and the oblique path of the grains,
- a coupling between the velocity field and the vertical diffusion: the Taylor-Aris dispersion

Dispersion in horizontal drum

$$c(x,t) = \frac{1}{\sqrt{4\pi D(\beta,\omega,h,...)t}} \exp\left(\frac{-(x - V_{ax}t)^2}{4D(\beta,\omega,h,...)t}\right)$$

$$w = \sqrt{DL}$$

Horizontal diffusion coefficient:



Taberlet PRE 2006

Axial dispersion in the rotating drum

Horizontal diffusion of cohesive grains

Spreading of impacting agglomerates

The potential energy: $$E_{\text{pot}} \sim \rho \phi d_{aggl}^3 g R$$

Breaking energy of the agglomerate: $$E_{\text{break}} \sim F_c Z \phi \frac{d_{aggl}^2}{d^2}$$

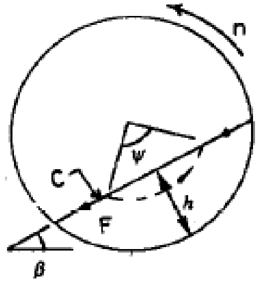
Large aggregates break $$d_{aggl} > \frac{F_c Z}{\rho g R d^2}$$

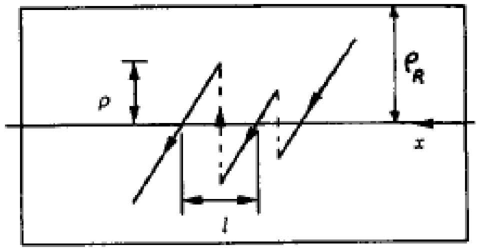
Influence of the geometry

Length of the avalanche $$l = 2y\left(\frac{\tan\beta}{\sin\theta_d} - \cot\theta_d \frac{\mathrm{d}h}{\mathrm{d}x}\right)$$

Period: $$\theta = \frac{2y}{\omega\left(y^2 + (R-h)^2\right)^{1/2}}$$

with a probability of P(*y*) $$P(y) = \frac{2y\,dy}{y_{min}^2}$$

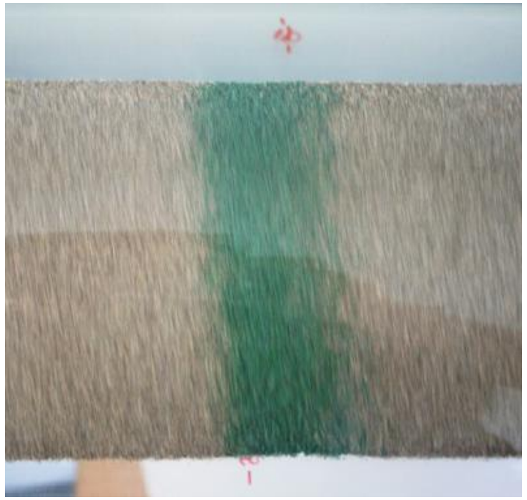


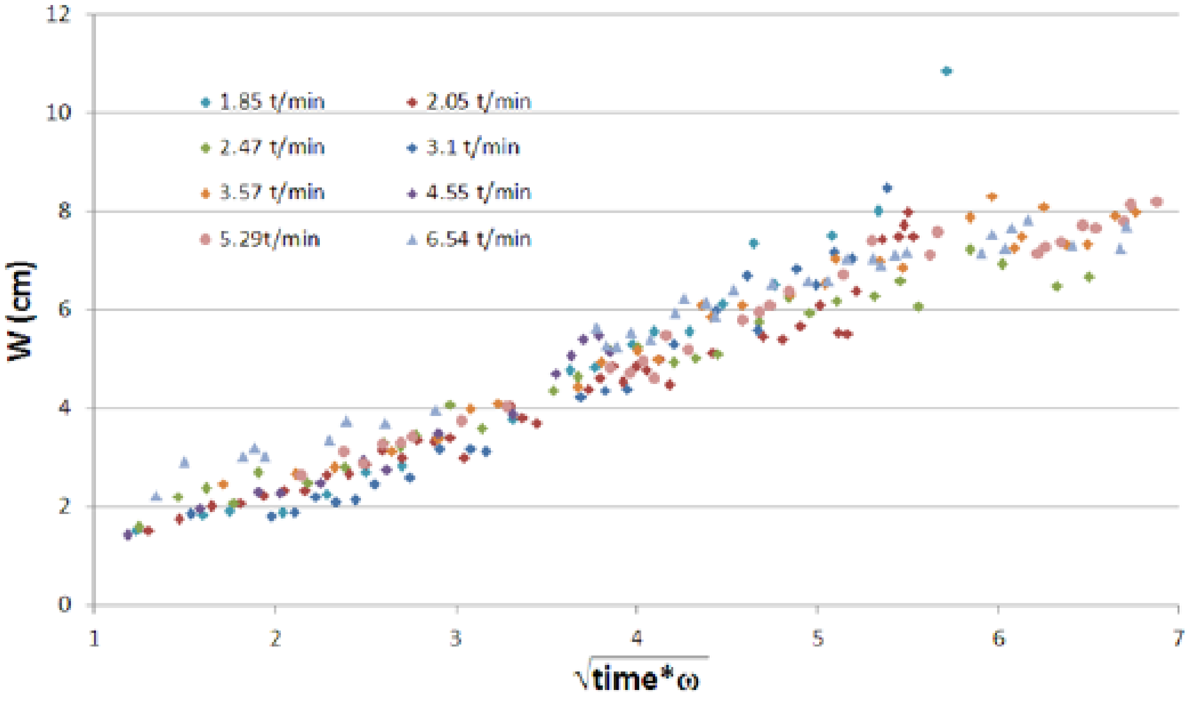


Dispersion coefficient: $$D_{hrz} = \frac{\langle l^2 \rangle}{2\langle \theta \rangle} \approx \frac{\omega y_{min}^3}{R f(h)}\left[\frac{\tan\beta}{\sin\theta_d} - \cot\theta_d\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)\right]^2$$

Fact checking: Experiments

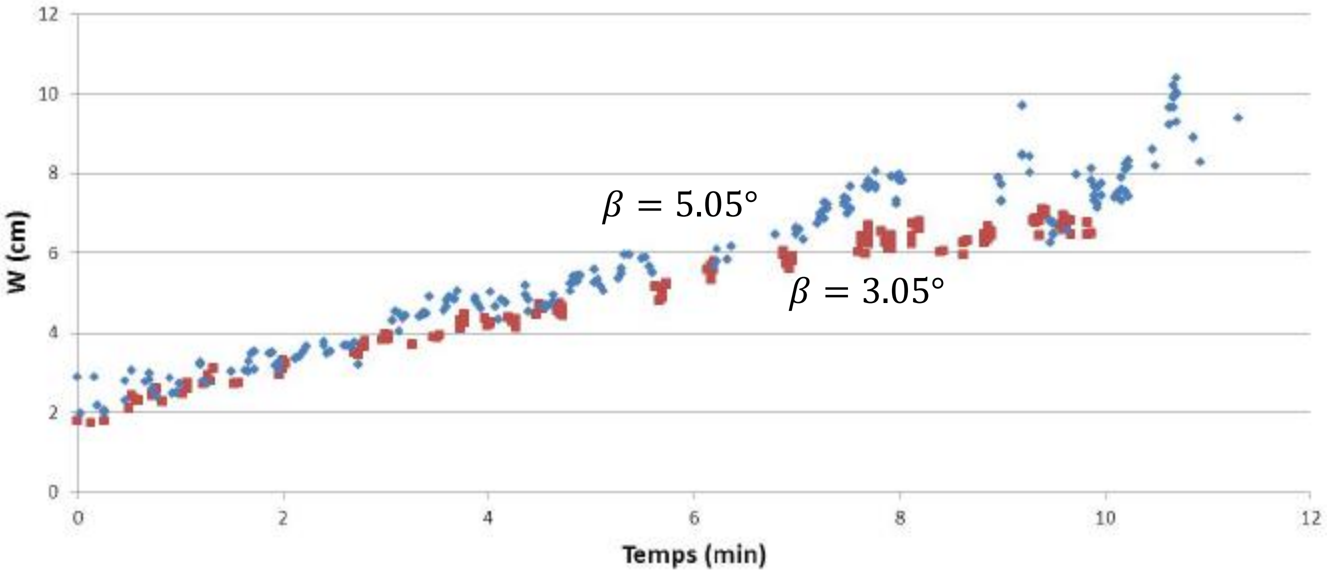
- The diffusion of colored grains in inclined cylinder:
  - The grains diffuse during the transport.
  - The diffusion is controlled by the rotation rate





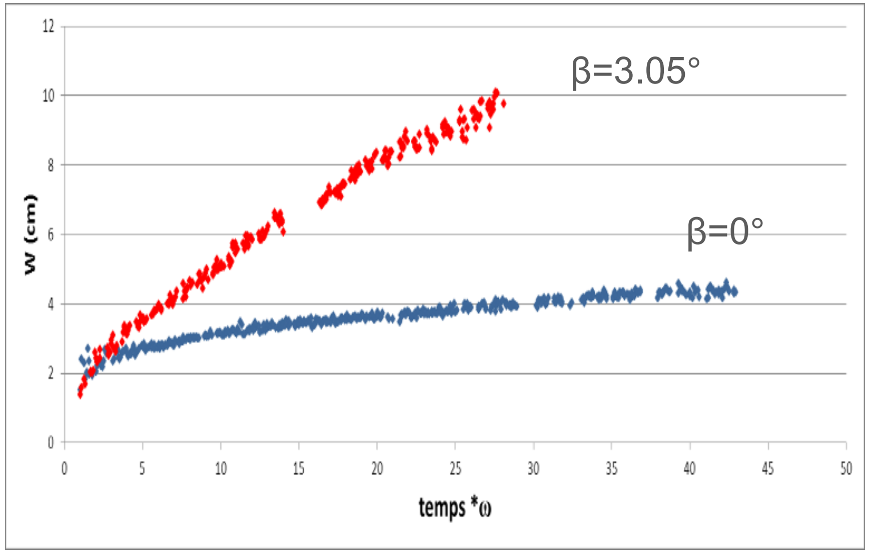
Role of the inclination

- Again, the diffusion is controlled by the time



$$W = \sqrt{\frac{D_{i0} \sin \beta_0}{\sin \beta_r} l}$$

But



- Diffusion is controlled by the time spent by grains in successive avalanches
  - But what is the underlying mechanism?
    - Only dispersion of granulates during avalanches in the surface layer?
    - Coupled with longitudinal transport?

Axial dispersion in the rotating drum

Taylor Aris dispersion

Concentration of the tracers

$$c(x,z,t) = \bar{c}(x,t) + c'(x,z,t)$$

$$V_x(z) = \bar{V} + V'_x(z)$$

Conservation of the tracers

$$\frac{\partial c}{\partial t} + V_x(z)\frac{\partial c}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial c}{\partial z}\right)$$

$$\frac{\partial \bar{c}}{\partial t} + \frac{\partial c'}{\partial t} + V_x\frac{\partial \bar{c}}{\partial x} + V'_x\frac{\partial \bar{c}}{\partial x} + V_x\frac{\partial c'}{\partial x} + V'_x\frac{\partial c'}{\partial x}$$
$$= \frac{\partial}{\partial x}\left(D\frac{\partial \bar{c}}{\partial x}\right) + \frac{\partial}{\partial x}\left(D\frac{\partial c'}{\partial x}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial \bar{c}}{\partial z}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial c'}{\partial z}\right)$$

$\bar{c} \gg c'$

$t \gg e^2/\bar{D} \quad l/\bar{V}_x \gg e^2/\bar{D}$

$$\frac{\partial \bar{c}}{\partial t} + \bar{V}_x\frac{\partial \bar{c}}{\partial x} + \overline{V'_x\frac{\partial c'}{\partial x}} \approx D\frac{\partial^2 \bar{c}}{\partial x^2}$$

Axial dispersion in the rotating drum

Taylor Aris dispersion

$$\bar{c} \gg c'$$

$$t \gg e^2/\bar{D} \qquad l/\bar{V}_x \gg e^2/\bar{D}$$

$$\frac{\partial c'}{\partial t} + \bar{V}_x \frac{\partial c'}{\partial x} + V'_x \frac{\partial \bar{c}}{\partial x} + V'_x \frac{\partial c'}{\partial x} - \overline{V'_x \frac{\partial c'}{\partial x}}$$
$$= \frac{\partial}{\partial x}\left[(D - \bar{D})\frac{\partial \bar{c}}{\partial x}\right] + D\frac{\partial^2 c'}{\partial x^2} + \frac{\partial}{\partial z}\left(D\frac{\partial c'}{\partial z}\right) - \frac{\partial}{\partial x}\left(\overline{D\frac{\partial c'}{\partial x}}\right)$$

$$\frac{\partial}{\partial z}\left(D\frac{\partial c'}{\partial z}\right) \approx V'_x \frac{\partial \bar{c}}{\partial x}$$

$$\frac{\partial \bar{c}}{\partial t} + \bar{V}_x \frac{\partial \bar{c}}{\partial x} + \overline{V'_x \frac{\partial c'}{\partial x}} \approx \bar{D}\frac{\partial^2 \bar{c}}{\partial x^2}$$

$$\overline{V'_x \frac{\partial c'}{\partial x}} = \frac{\partial^2 \bar{c}}{\partial x^2}\left[\overline{V'_x(z)\int_0^z \frac{1}{D}\int_0^{z_1} V'_x(z_2)\,dz_2\,dz_1}\right] = D_{shr}\frac{\partial^2 \bar{c}}{\partial x^2}$$

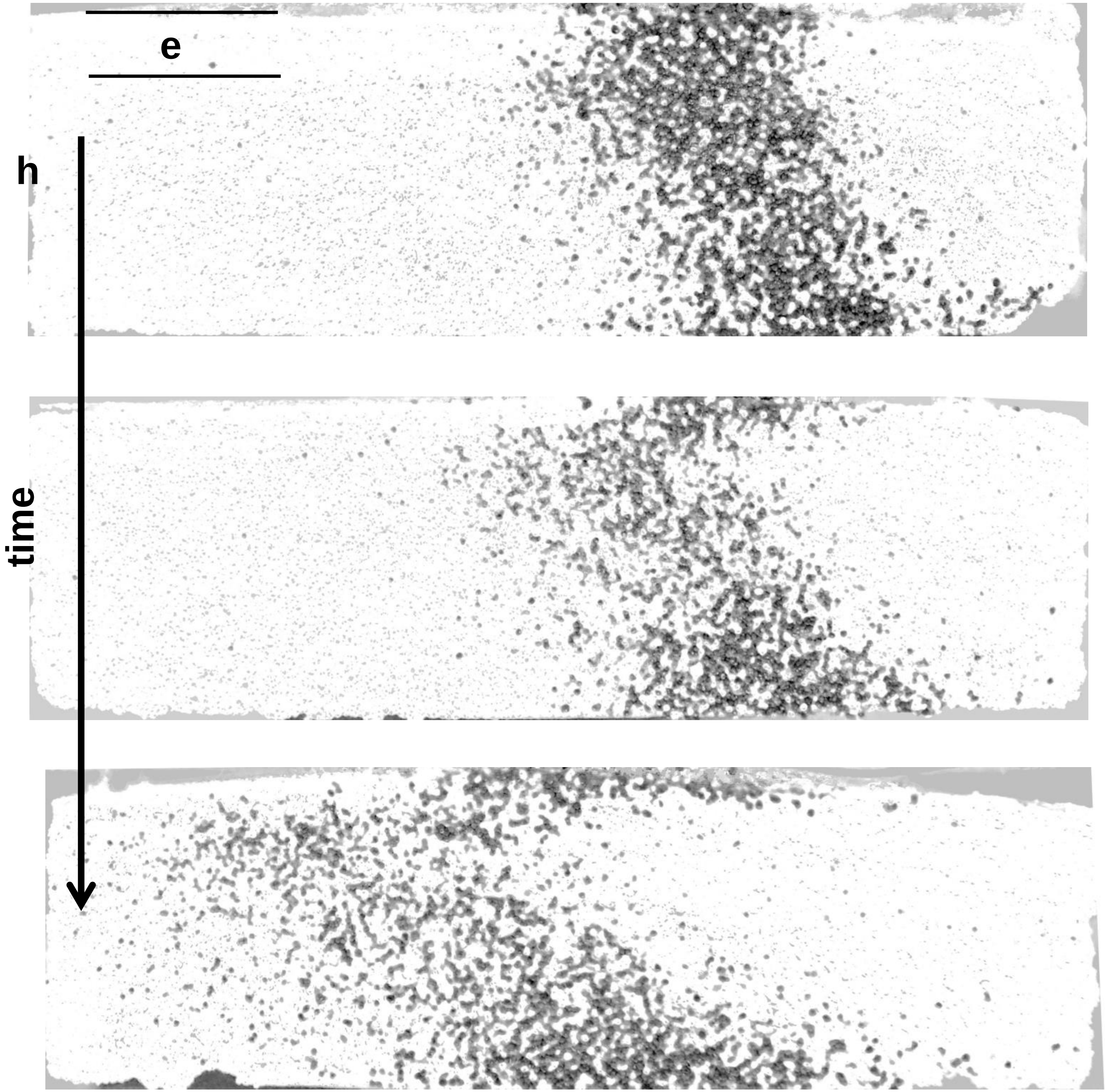
$$\frac{\partial \bar{c}}{\partial t} + \bar{V}_x \frac{\partial \bar{c}}{\partial x} = D_{eff}\frac{\partial^2 \bar{c}}{\partial x^2},$$

with, for a linear shear:

$$D_{eff} = D_{hrz}\left[1 + \frac{\dot{\gamma}^2 e^4}{30 D_{hrz}^2}\right] = D_{hrz}\left[1 + \frac{Pe^2}{30}\right]$$

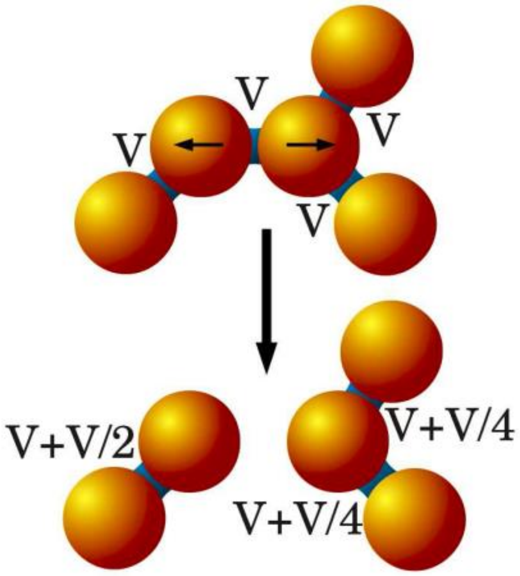
$$Pe \approx \frac{\dot{\gamma}}{\dot{\gamma}_y}\frac{e^2}{d^2}$$

Christov Gran. Matter 2014


e
h
time

Spreading the liquid by shearing the grains

- **Rupture of the bridges in 2 identical volumes**
- Swelling of the thin films that allows a fast redistribution according the pressure gradients $\Delta V_i = (\gamma/r - P_i) / L_i$

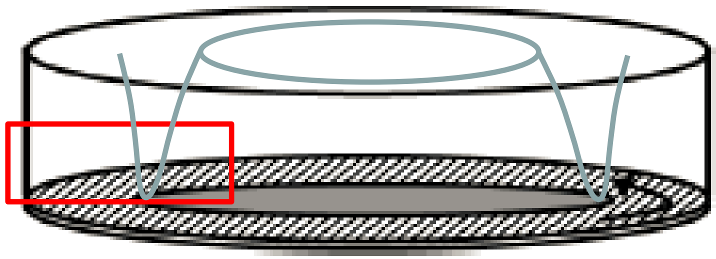


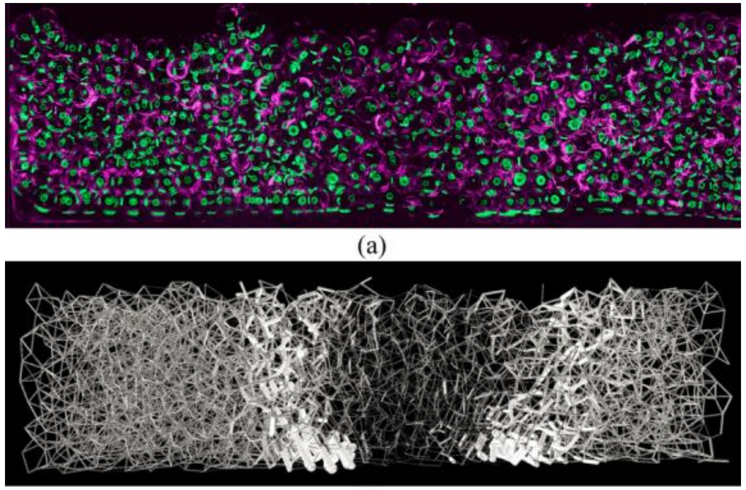
- **Creation of a bridge**
- **Viscous relaxation toward equilibrium**

$\partial_t V_i = \mu \Sigma_j (Pi - Pj)$

  - it takes some times,

⇒ Steady state depending on the shear rate





(a)

(b)

Mani PRL 2012

Diffusion of the volume of the liquid bridges

Flux of liquid in a slice of material

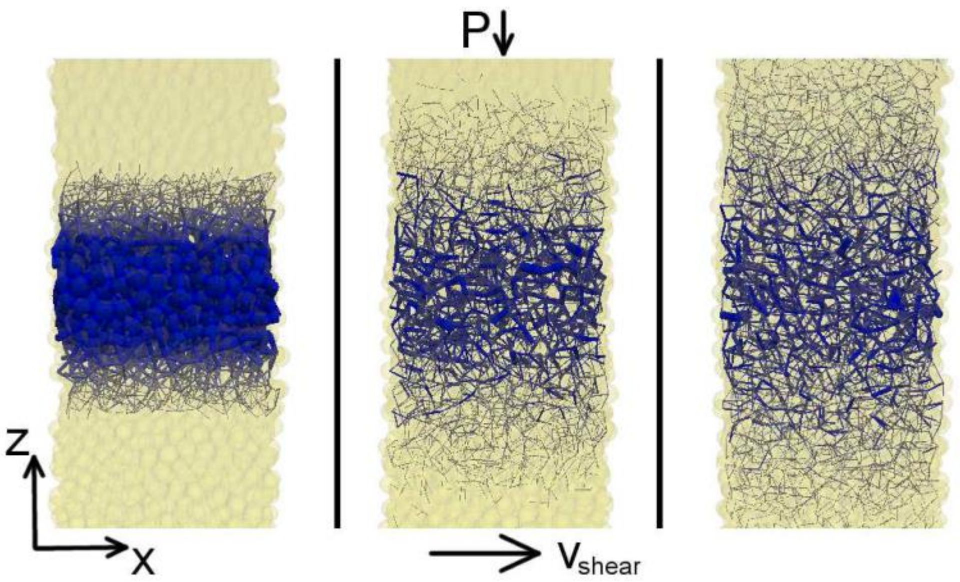
$Q^i(t+dt)-Q^i(t) = Adt\,(B^{i-1}Q_b^{i-1} + B^{i+1}Q_b^{i+1} - 2B^iQ_b^i)/2$

B: rate of rupture
Q_b: average volume per bridge

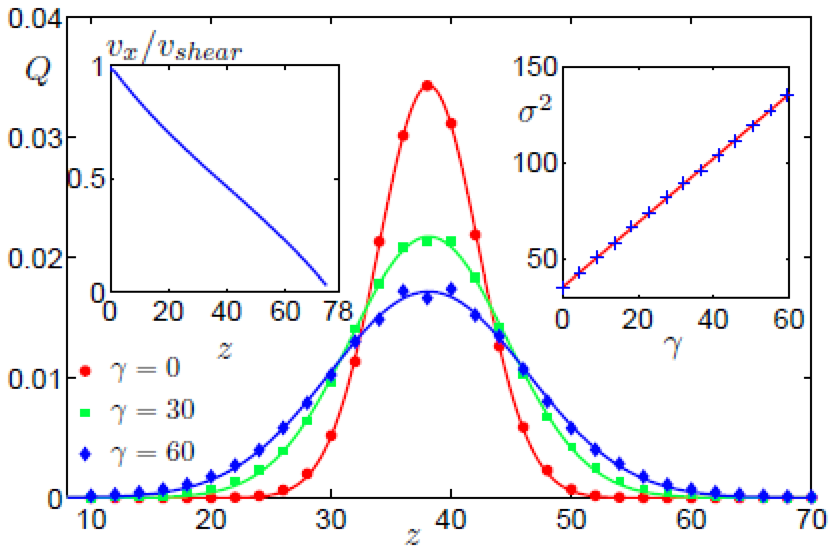
- "Diffusion" like equation

$$\partial_t Q_b = C \frac{\partial}{\partial z^2}(\dot{\gamma} Q_b)$$

Neglecting the cohesion (P>>1, I<<1), linear velocity profile

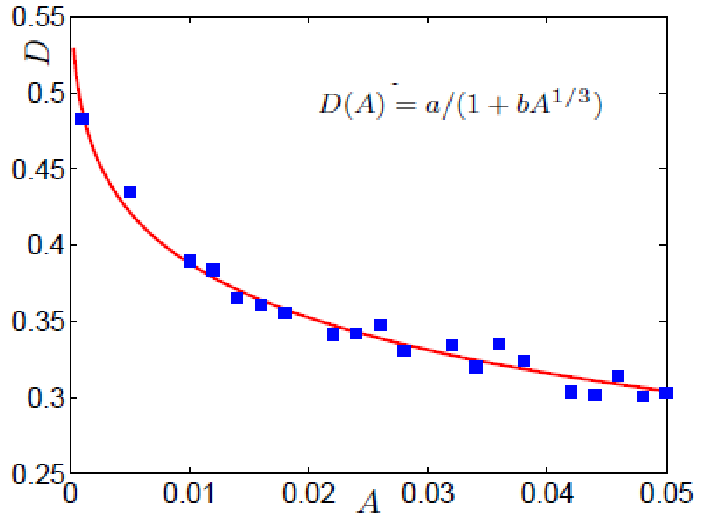


Mani PRL 2012



Influence of the amount of liquid on the spreading

➡ **the amplitude of the initial distribution A**



- $A \propto V \approx Q$
- Rupture distance: $s_c \approx V^{1/3}$

- The time for rupture increases with s_c :
- $T \propto \dot{\gamma}^{-1}(1 + s_c/r)$

➡ **Application :**

- Coating of grains,

e.g. closing open porosity with a polymer on beads

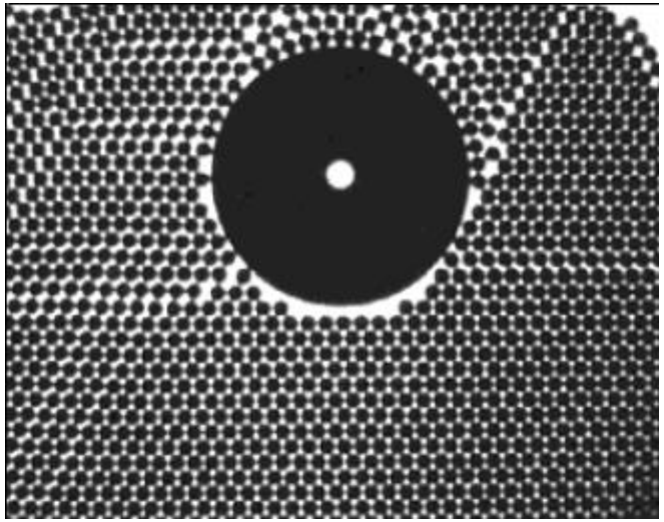
Dispersion in granular media

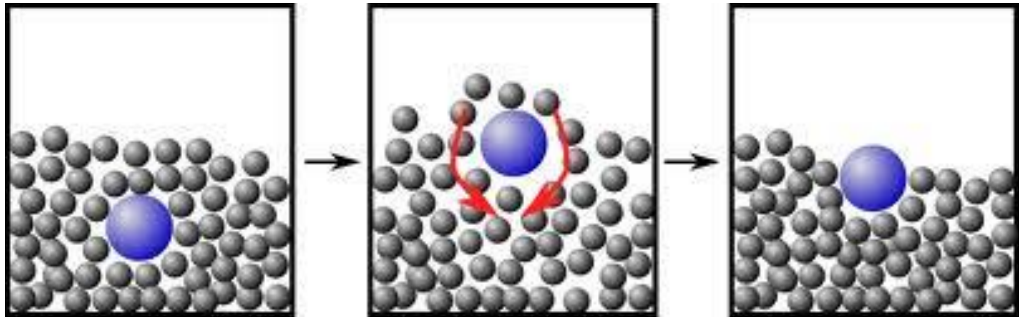
- **Granular media**
- **Rheology**
- **Diffusion**
- **Segregation**

Steric effect

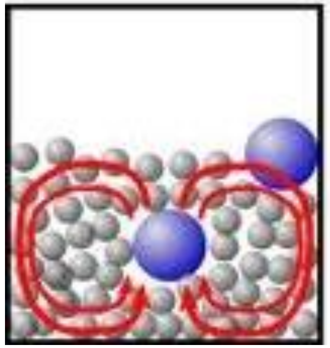
Large grains can not fill the space

But the small ones



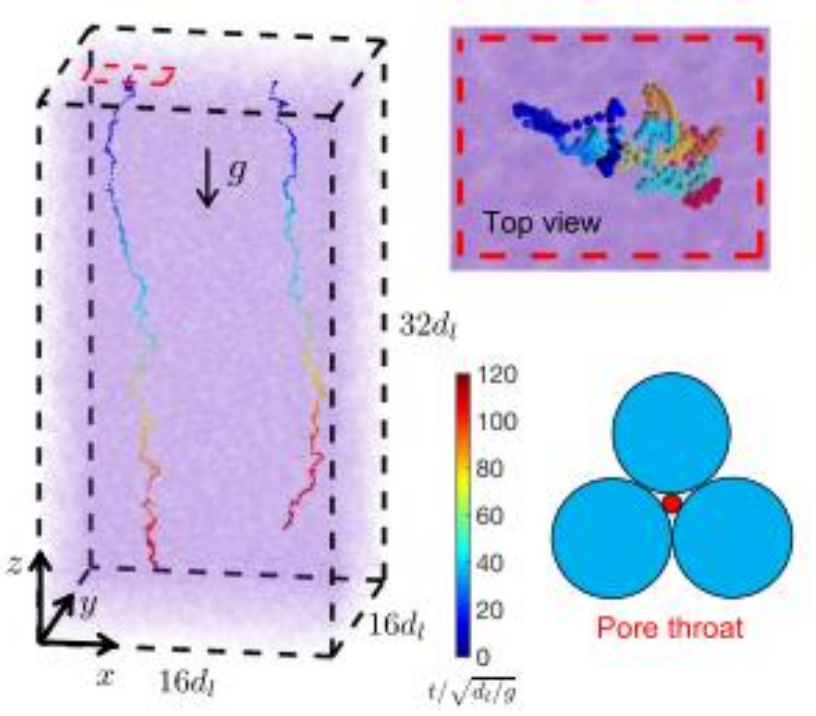


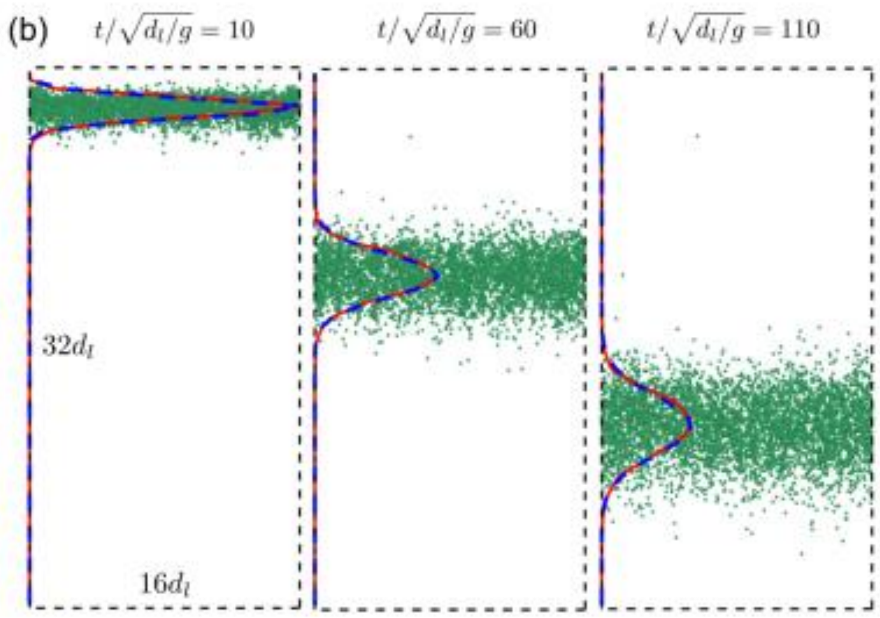
+ convection

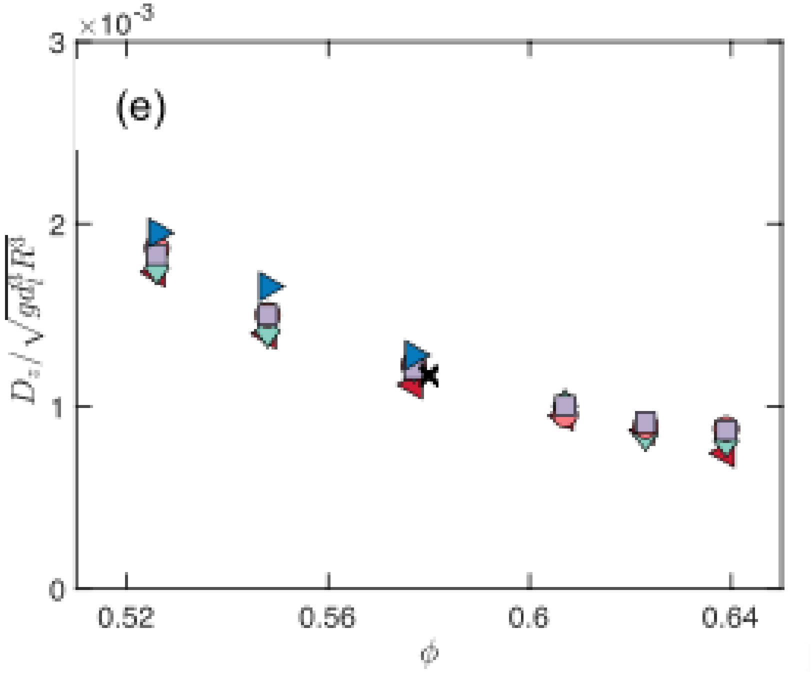


Segregation

Percolation in a static media



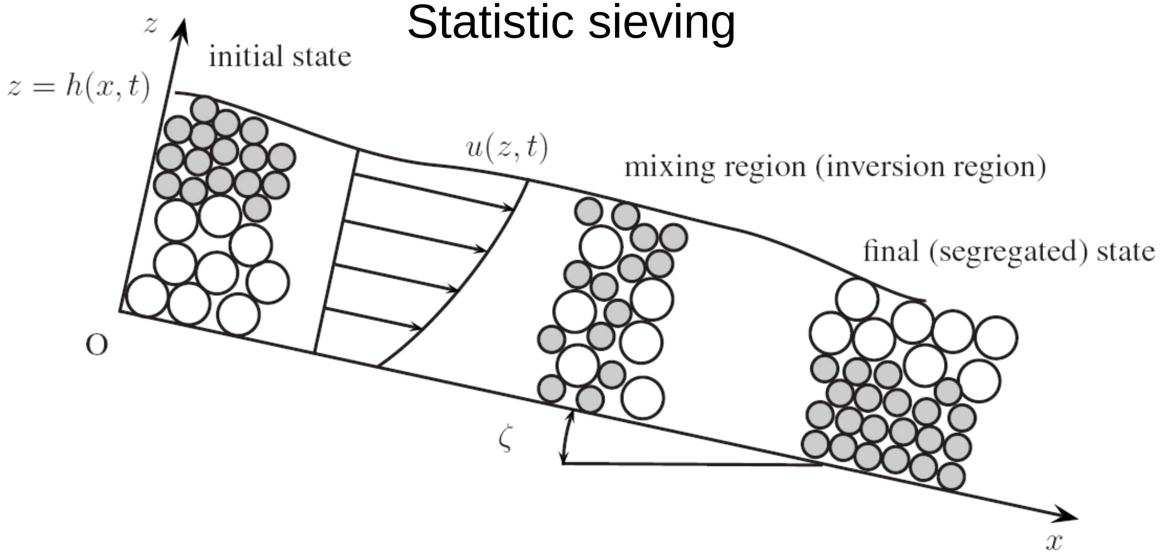




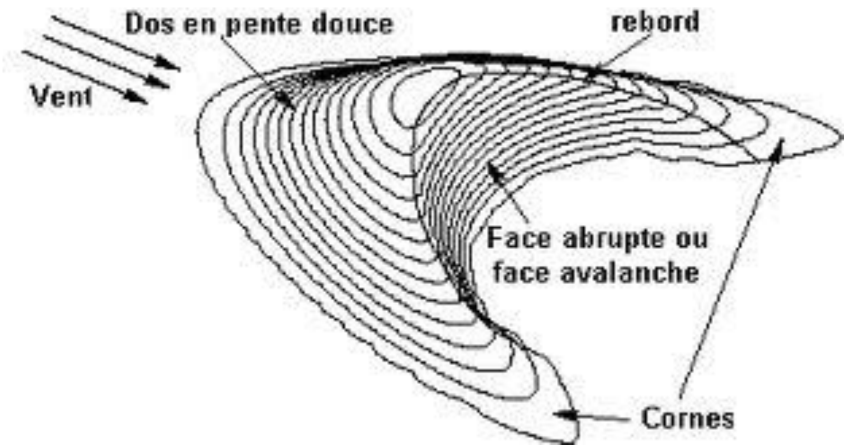
Gao PRE 2023

Segregation during a flow

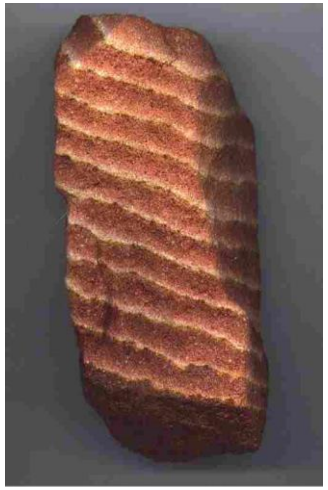
Statistic sieving



Avalanches on dunes

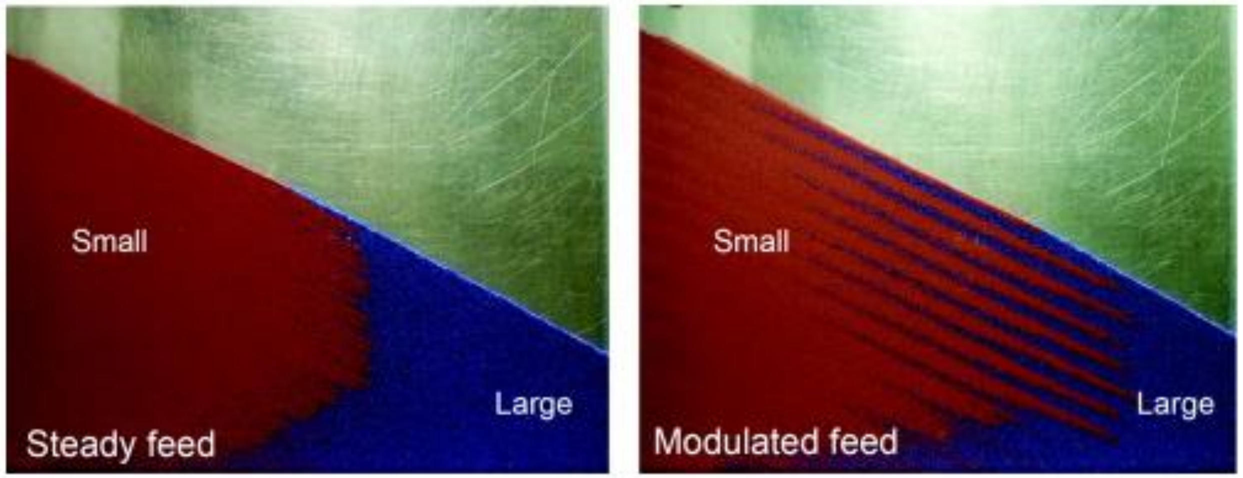


Pétra stone



Segregation in pile flow

Interplay with the flow rate and the segregation



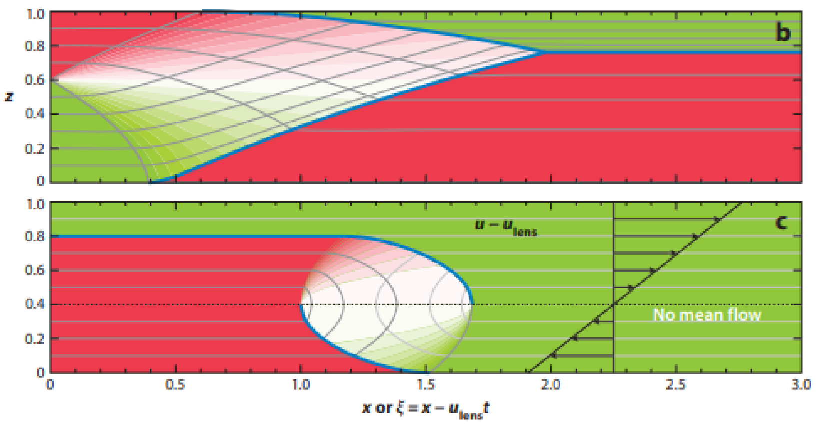
Xiao Powders Tech. 2027

Saint-Venant approach:

$$\frac{\partial c_i}{\partial t} + \underbrace{\frac{\partial (u c_i)}{\partial x} + \frac{\partial (w c_i)}{\partial z}}_{\text{advection}} + \underbrace{\frac{\partial (w_{p,i} c_i)}{\partial z}}_{\text{segregation}} - \underbrace{\left[\frac{\partial}{\partial x}\left(D\frac{\partial c_i}{\partial x}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial c_i}{\partial z}\right)\right]}_{\text{diffusion}} = 0$$

$$w_{p,s} = -S\dot{\gamma}(1 - c_s)$$

$$D \sim \dot{\gamma} d^2$$

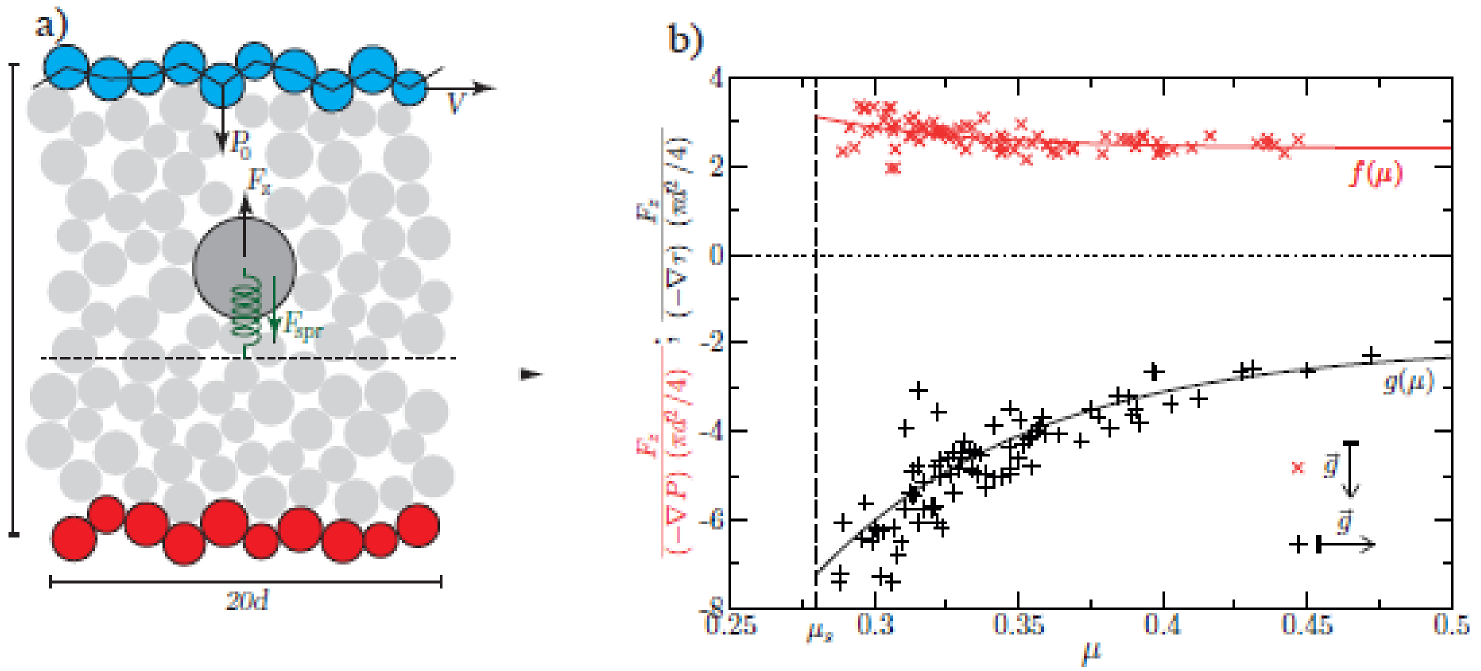


Thronton & Gray JFM 2006,2008

Segregation in a shear flow

Interplay with the flow rate and the segregation

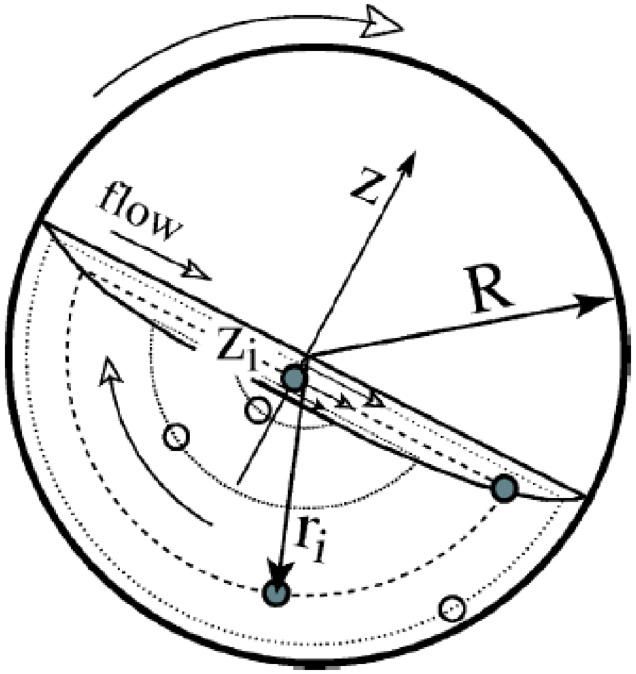
Mechanical approach:

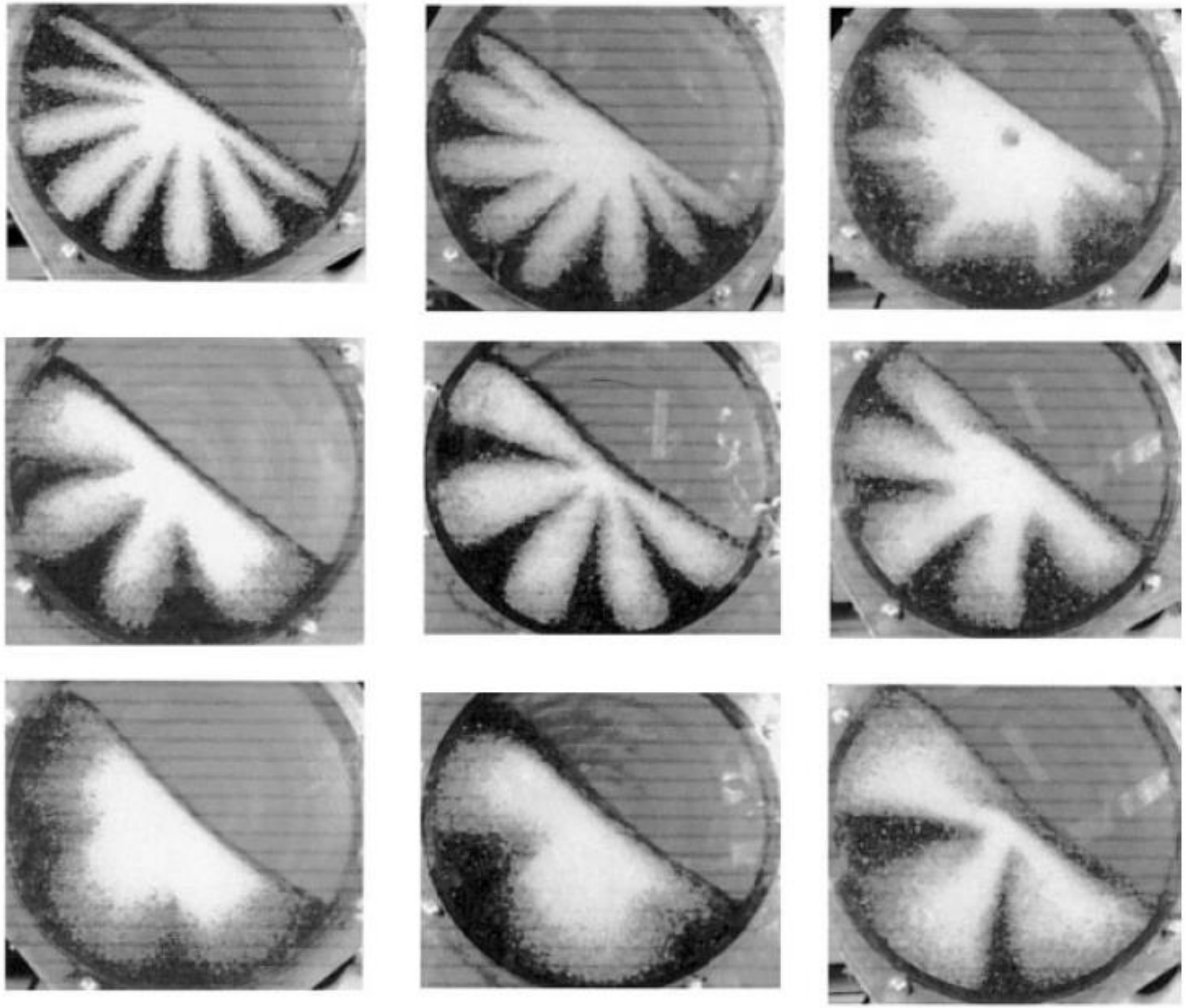


Guillard JFM 2016

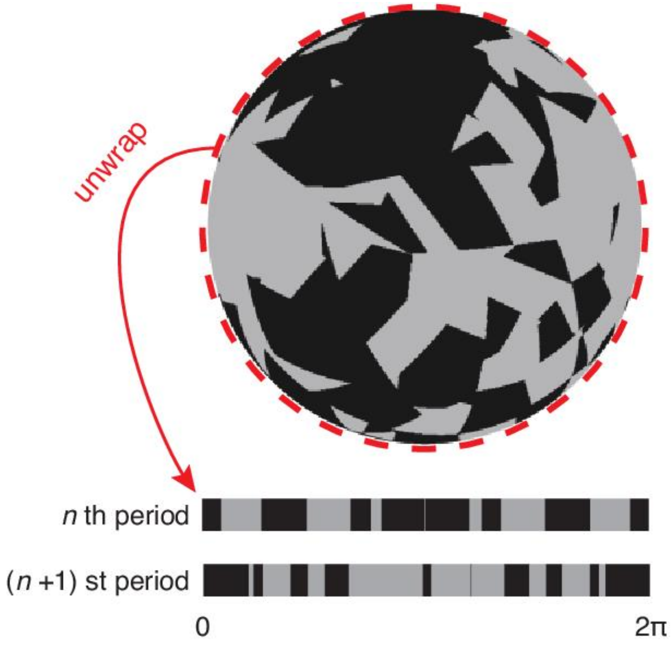
Segregation in rotating drum

Equilibrium position from a balance between buoyancy and weight

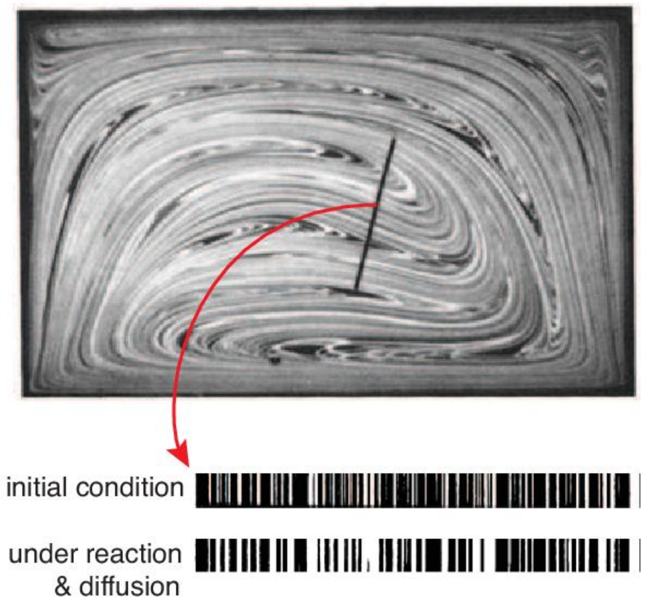




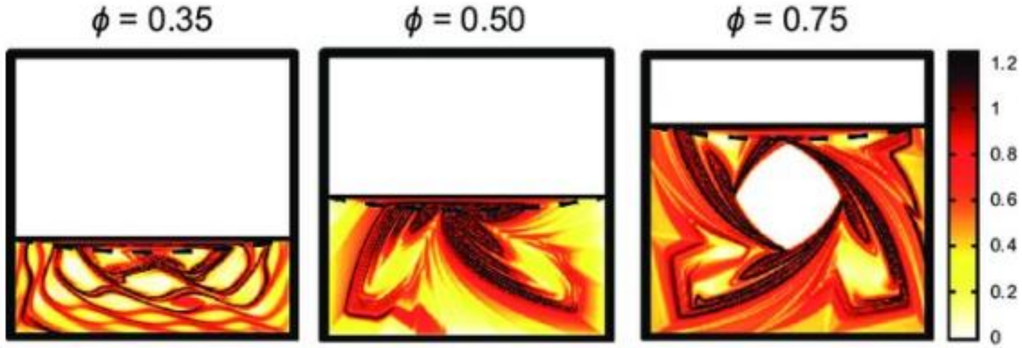
Cutting and shuffling strategy / streamlines jumping



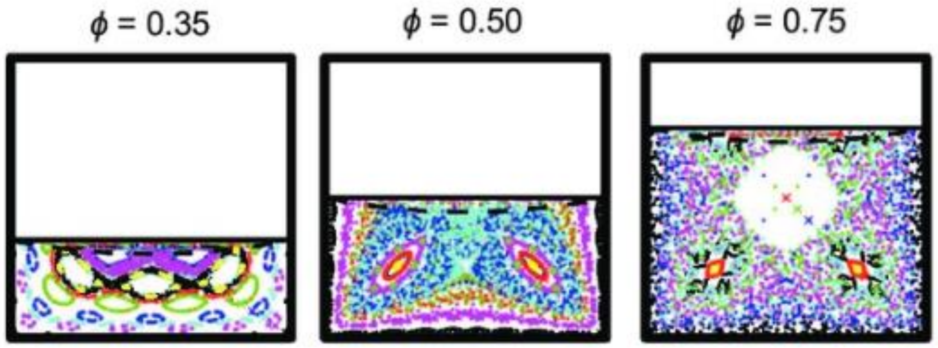
(a) bottom view of a granular mixing PWI simulation in a sphere



(b) fluid mixing experiment in a cavity with moving top wall

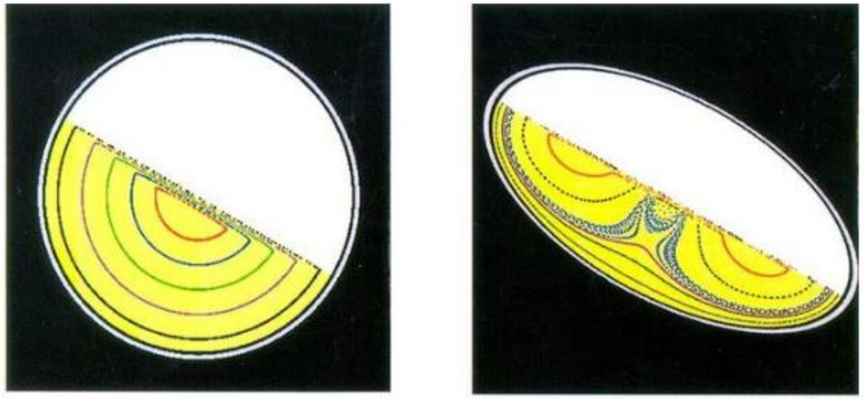


(a) FTLE field with respect to $t = -14T$



(b) Poincaré section over 500 periods

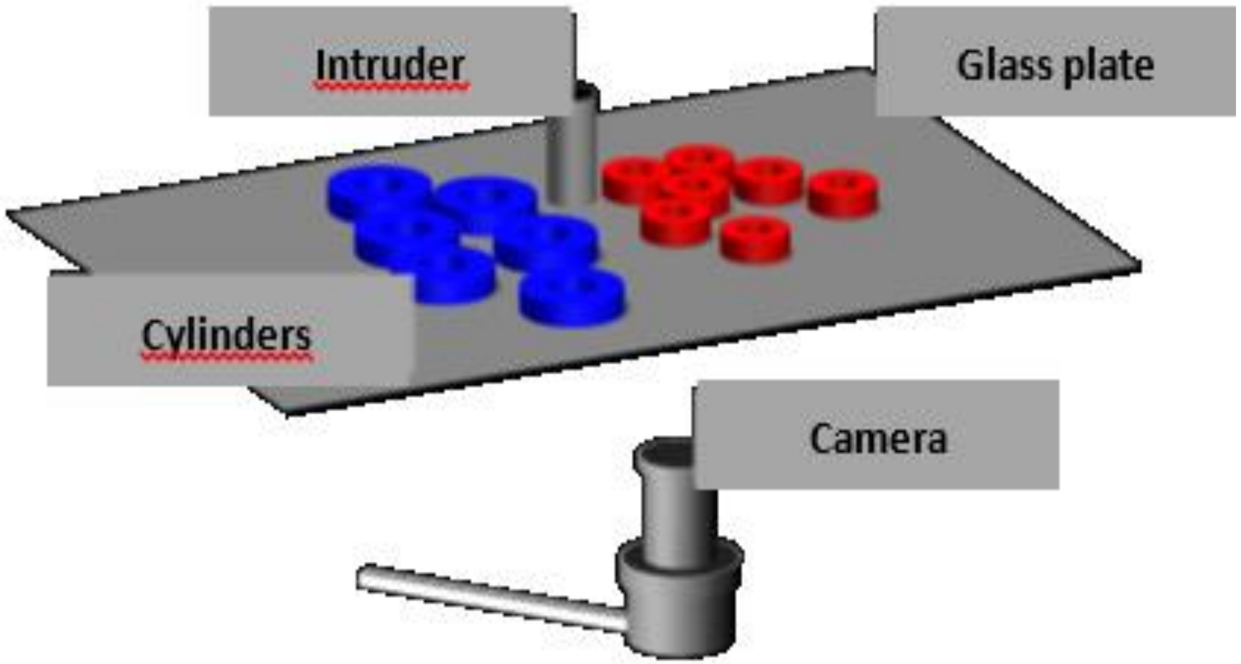
Christov POF 2011

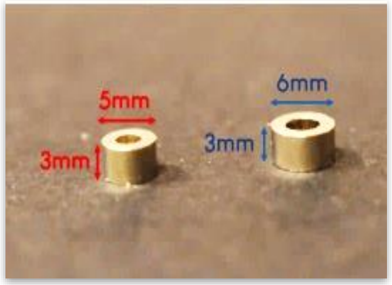


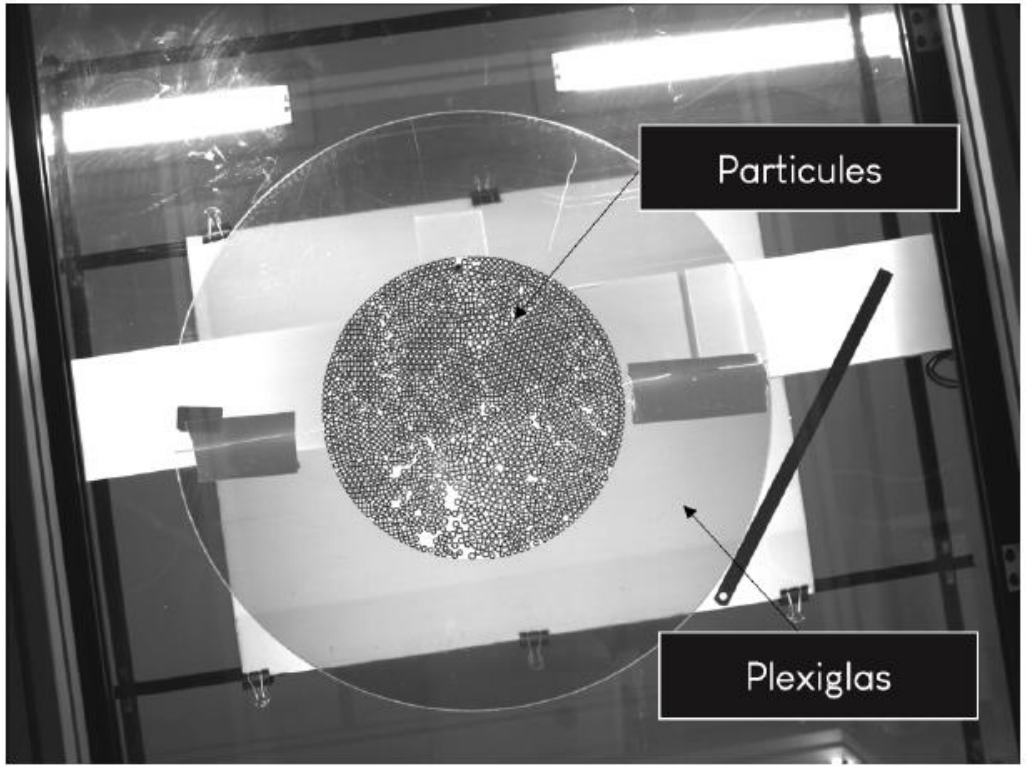
KAM islands, FTLE, …

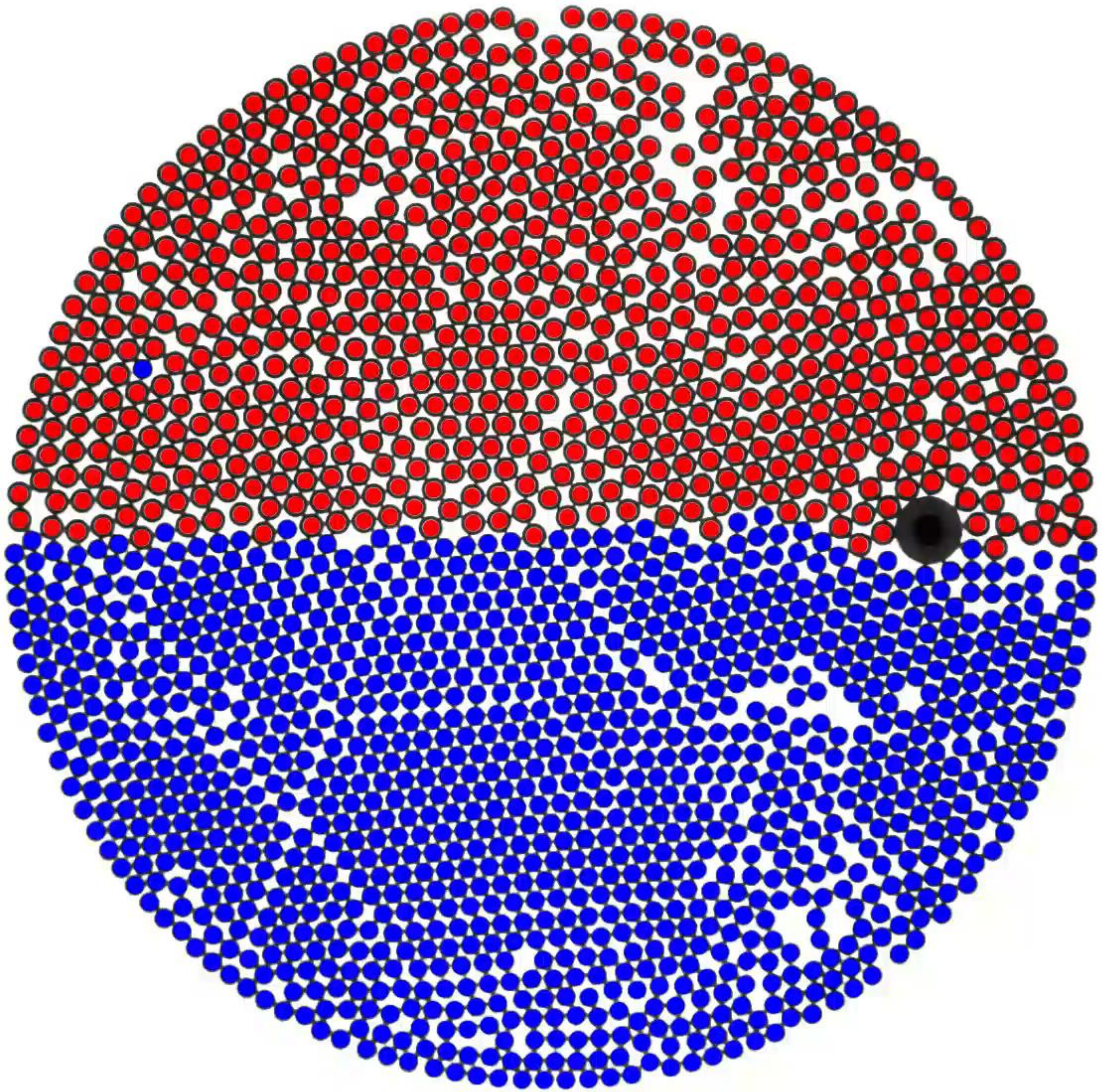
MIXING IN A 2D GRANULAR SYSTEMS :

2D SYSTEMS

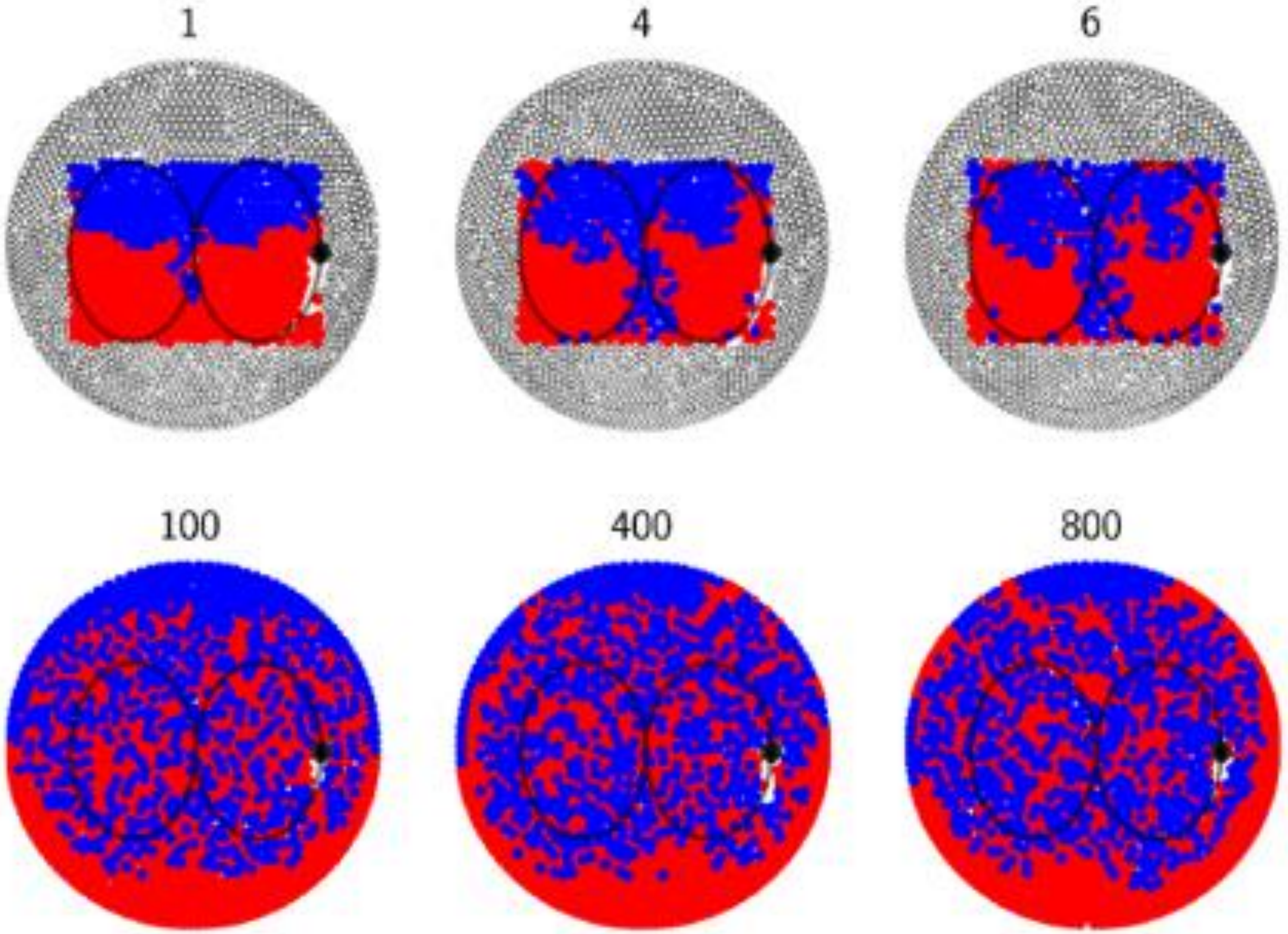




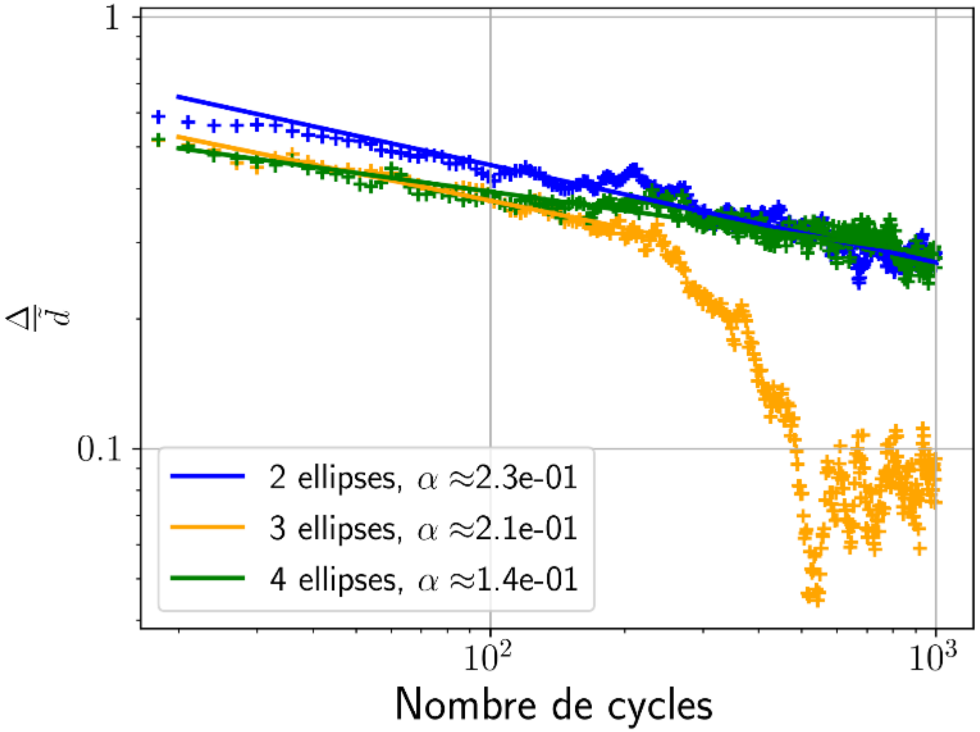


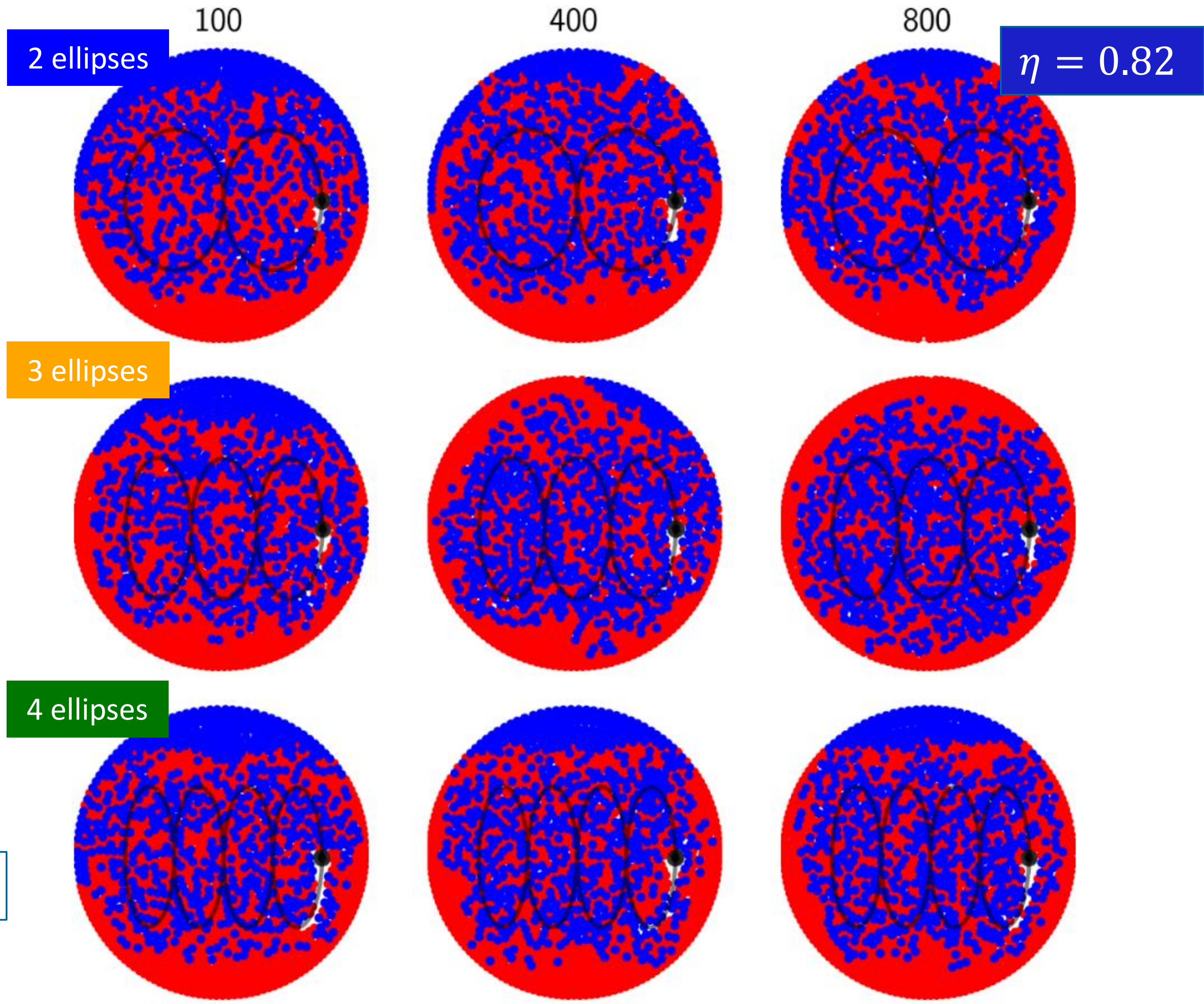


Example of large (blue) and small (red) segregated system that tends to homogenise with the number of



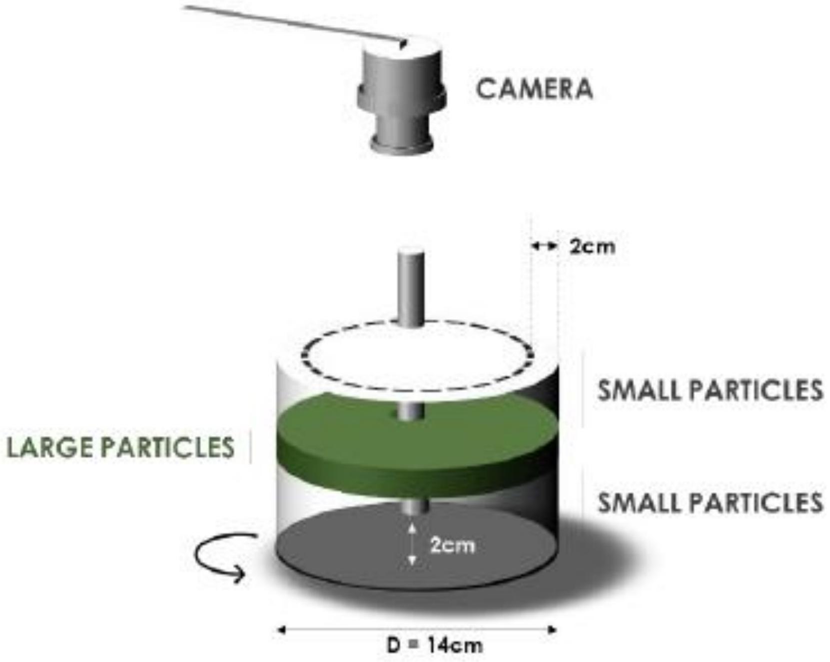
ROLE OF THE VELOCITY FIELD

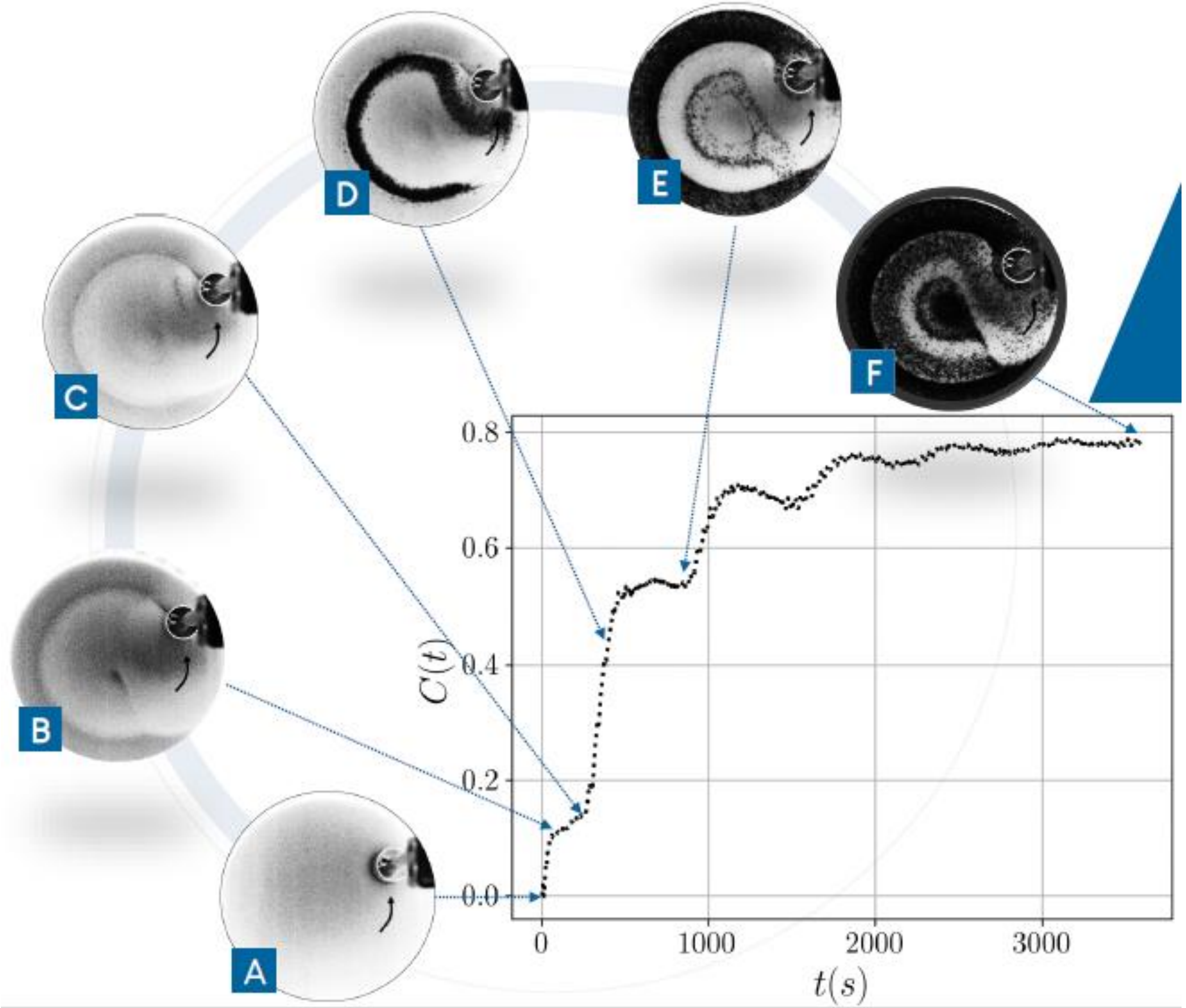




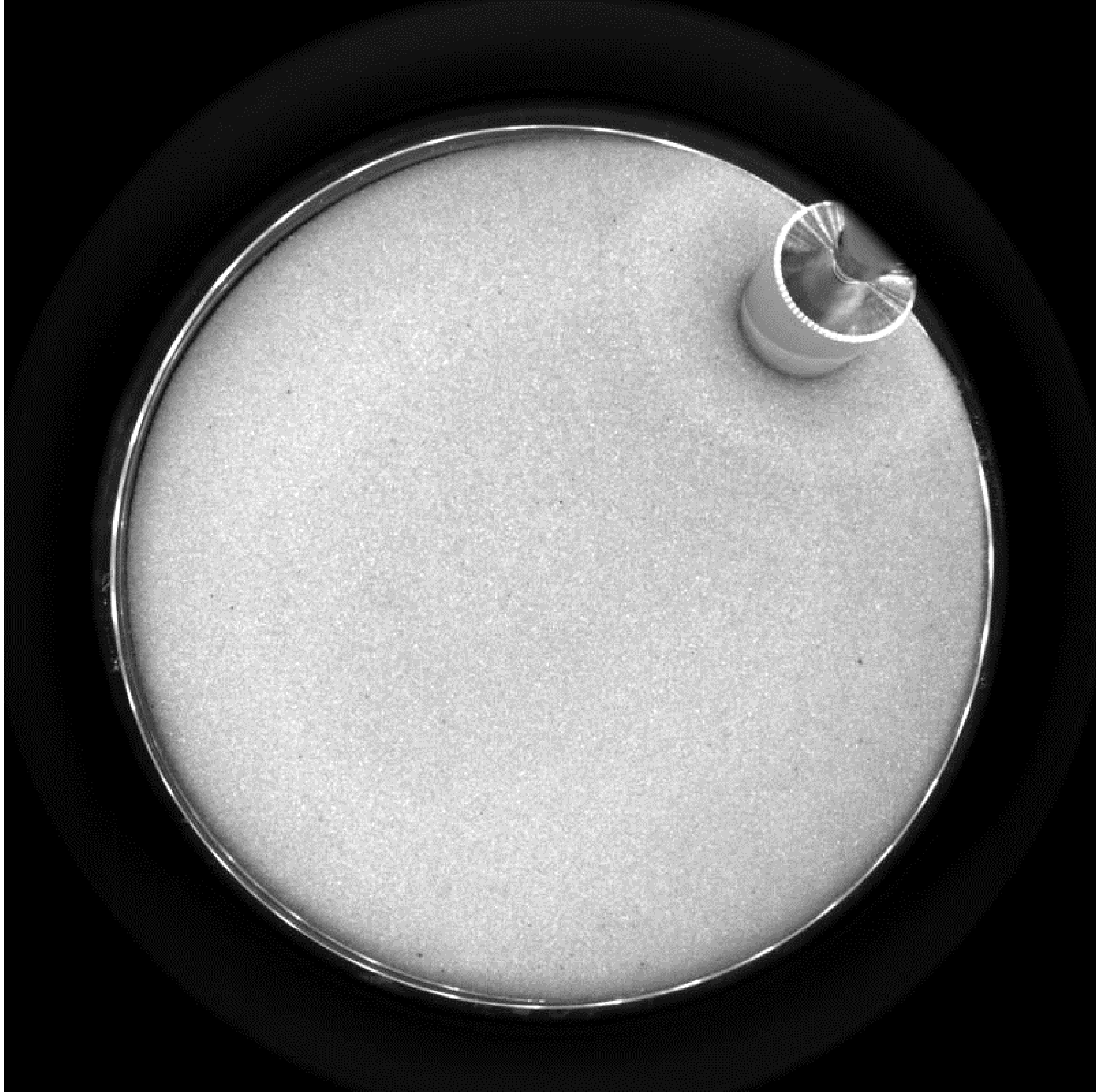
3 ellipses more efficient close to the boundary

SEGREGATION

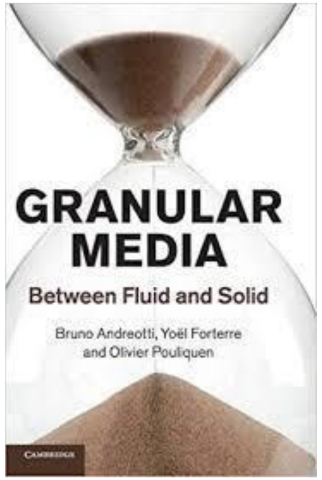




Mayeden EPJ 2021



Reference :
Andreotti B., Forterre Y., Pouliquen O. (2013) GranularMedia; BetweenFluidand Solid, Cambridge UniversityPress



Take home messages

- **Dispersion in granular media is coupled with the flow**
- **Granular media can be the substrate for other processes (chemical reactions, …)**
- **Homogeneization processes should often conteract segregation phenomonon**
- **Smooth complex flows are much less studied in the context of « mixing ».**

Mixing in
granular media
Jop’s lecture

Daniel

Howard